

Contents

Preface	vii
Chapter 1 Examples of Quantizations	1
1.1 Quantization of \mathbf{R}^2	1
1.1.1 Classical mechanics	1
1.1.2 Symplectic method	3
1.1.3 Holomorphic method	6
1.2 Holomorphic representation of symplectic quotients and its quantization	7
1.2.1 An example of circle action	7
1.2.2 Moment map of symplectic actions	9
1.2.3 Some geometric invariant theory	11
1.2.4 Grassmanians	12
1.2.5 Calabi-Yau/Ginzburg-Landau correspondence	13
1.2.6 Quantization of symplectic quotients	14
Chapter 2 Classical Solutions of Gauge Field Theory	17
2.1 Moduli space of classical solutions of Chern-Simons action	17
2.1.1 Symplectic reduction of gauge fields over a Riemann surface	17
2.1.2 Chern-Simons action on a three manifold	19
2.2 Maxwell equations and Yang-Mills equations	22
2.2.1 Maxwell equations	22
2.2.2 Yang-Mills equations	23
2.3 Vector bundle, Chern classes and Chern-Weil theory	25
2.3.1 Vector bundle and connection	25

2.3.2	Curvature, Chern classes and Chern-Weil theory	26
-------	--	----

Chapter 3 Quantization of Chern-Simons Action **27**

3.1	Introduction	27
3.2	Some formal discussions on quantization	28
3.3	Pre-quantization	31
3.3.1	\mathcal{M} as a complex variety	31
3.3.2	Quillen's determinant bundle on \mathcal{M} and the Laplacian .	32
3.4	Some Lie groups	32
3.4.1	$G = \mathbf{R}$	32
3.4.2	$G = S^1 = \mathbf{R}/2\pi\mathbf{Z}$	33
3.4.3	T^*G	34
3.5	Compact Lie groups, $G = SU(2)$	35
3.5.1	Genus one	35
3.5.2	Riemann sphere with punctures	36
3.5.3	Higher genus Riemann surface	38
3.5.4	Relation with WZW model and conformal field theory .	39
3.6	Independence of complex structures	40
3.7	Borel-Weil-Bott theorem of representation of Lie groups	44

Chapter 4 Chern-Simons-Witten Theory and Three Manifold Invariant **47**

4.1	Representation of mapping class group and three manifold invariant	47
4.1.1	Knizhik-Zamolodchikov equations and conformal blocks	48
4.1.2	Braiding and fusing matrices	50
4.1.3	Projective representation of mapping class group	53
4.1.4	Three-dimensional manifold invariants via Heegard decomposition	57
4.2	Calculations by topological quantum field theory	59
4.2.1	Atiyah's axioms	59
4.2.2	An example: connected sum	60
4.2.3	Jones polynomials	60
4.2.4	Surgery	61
4.2.5	Verlinde's conjecture and its proof	63
4.3	A brief survey on quantum group method	64
4.3.1	Algebraic representation of knot	64
4.3.2	Hopf algebra and quantum groups	67

4.3.3	Chern-Simons theory and quantum groups	68
Chapter 5 Renormalized Perturbation Series of Chern-Simons-Witten Theory		71
5.1	Path integral and morphism of Hilbert spaces	71
5.1.1	One-dimensional quantum field theory	71
5.1.2	Schroedinger operator	72
5.1.3	Spectrum and determinant	75
5.2	Asymptotic expansion and Feynman diagrams	77
5.2.1	Asymptotic expansion of integrals, finite dimensional case	77
5.2.2	Integration on a sub-variety	81
5.3	Partition function and topological invariants	82
5.3.1	Gauge fixing and Faddeev-Popov ghosts	83
5.3.2	The leading term	85
5.3.3	Wilson line and link invariants	88
5.4	A brief introduction on renormalization of Chern-Simons theory	89
5.4.1	A regularization scheme	90
5.4.2	The Feynman rules	91
Chapter 6 Topological Sigma Model and Localization		95
6.1	Constructing knot invariants from open string theory	95
6.1.1	Introduction	95
6.1.2	A topological sigma model	96
6.1.3	Localization principle	97
6.1.4	Large N expansion of Chern-Simons gauge theory	98
6.2	Equivariant cohomology and localization	99
6.2.1	Equivariant cohomology	99
6.2.2	Localization, finite dimensional case	100
6.3	Atiyah-Bott's residue formula and Duistermaat-Heckman formula	101
6.3.1	Complex case, Atiyah-Bott's residue formula	101
6.3.2	Symplectic case, Duistermaat-Heckman formula	102
6.4	2D Yang-Mills theory by localization principle	104
6.4.1	Cohomological Yang-Mills field theory	104
6.4.2	Relation with physical Yang-Mills theory	105
6.4.3	Evaluation of Yang-Mills theory	107
6.5	Combinatorial approach to 2D Yang-Mills theory	110

Complex Manifold Without Potential Theory by S. S. Chern	113
Geometric Quantization of Chern-Simons Gauge Theory by S. Axelrod, S. D. Pietra and E. Witten	121
On Holomorphic Factorization of WZW and Coset Models	169
Bibliography	193
Index	197
Afterwards	199