

# Contents

<b>Introduction</b>	<b>v</b>
<b>Acknowledgments</b>	<b>vii</b>
<b>1 Groups and Geometry</b>	<b>1</b>
1.1 Before the Lab . . . . .	1
1.2 In the Lab . . . . .	6
1.3 Further Work . . . . .	7
1.4 Geometric Shapes . . . . .	7
<b>2 Cayley Tables</b>	<b>9</b>
2.1 Binary Operations on Sets . . . . .	9
2.1.1 Before the Lab . . . . .	9
2.1.2 In the Lab . . . . .	10
2.2 Cayley Tables for Groups of Orders 2, 3 and 4 . . . . .	11
2.2.1 Before the Lab . . . . .	11
2.2.2 In the Lab . . . . .	12
<b>3 Cyclic Groups and Cyclic Subgroups</b>	<b>15</b>
3.1 Before the Lab . . . . .	15
3.2 In the Lab . . . . .	17
3.3 Further Work . . . . .	19
<b>4 Subgroups and Subgroup Lattices</b>	<b>21</b>
4.1 Before the Lab . . . . .	21
4.2 In the Lab . . . . .	23
4.3 Further Work . . . . .	24

<b>5</b>	<b>The Center and Commutator Subgroups</b>	<b>27</b>
5.1	Before the Lab . . . . .	27
5.2	In the Lab . . . . .	28
5.3	Further Work . . . . .	29
<b>6</b>	<b>Quotient Groups</b>	<b>31</b>
6.1	Before the Lab . . . . .	31
6.2	In the Lab . . . . .	34
6.3	Further Work . . . . .	35
<b>7</b>	<b>Direct Products</b>	<b>37</b>
7.1	Before the Lab . . . . .	37
7.1.1	External Direct Products . . . . .	37
7.1.2	Internal Direct Products . . . . .	38
7.2	In the Lab . . . . .	40
7.3	Further Work . . . . .	40
<b>8</b>	<b>The Unitary Groups</b>	<b>41</b>
8.1	Before the Lab . . . . .	41
8.2	In the Lab . . . . .	42
8.3	Further Work . . . . .	43
<b>9</b>	<b>Composition Series</b>	<b>45</b>
9.1	Before the Lab . . . . .	45
9.2	In the Lab . . . . .	47
9.3	Further Work . . . . .	48
<b>10</b>	<b>Introduction to Endomorphisms</b>	<b>49</b>
10.1	Before the Lab . . . . .	49
10.2	In the Lab . . . . .	51
<b>11</b>	<b>The Inner Automorphisms of a Group</b>	<b>55</b>
11.1	Before the Lab . . . . .	55
11.2	In the Lab . . . . .	56
11.3	Further Work . . . . .	57
<b>12</b>	<b>The Kernel of an Endomorphism</b>	<b>59</b>
12.1	Before the Lab . . . . .	59
12.2	In the Lab . . . . .	60
12.3	Further Work . . . . .	62

<b>13 The Class Equation</b>	<b>63</b>
13.1 Before the Lab . . . . .	63
13.2 In the Lab . . . . .	64
13.3 Further Work . . . . .	65
<b>14 Conjugate Subgroups</b>	<b>67</b>
14.1 Before the Lab . . . . .	67
14.2 In the Lab . . . . .	68
14.3 Further Work . . . . .	69
<b>15 The Sylow Theorems</b>	<b>71</b>
15.1 Before the Lab . . . . .	71
15.2 In the Lab . . . . .	72
15.3 Further Work . . . . .	74
<b>Appendix A</b> Table Generation Menu of <i>ESG</i>	75
<b>Appendix B</b> Sample Library of <i>ESG</i>	77
<b>Appendix C</b> Group Library of <i>ESG</i>	79
<b>Appendix D</b> Group Properties Menu	81