

# Contents

Preface	ix
Acknowledgments	xiii
Chapter 1. Differential equations and their geometry	1
1. The Cauchy problem for first-order partial differential equations	1
2. Hyperbolic equations integrable by the method of Darboux	3
3. External, internal and generalized symmetries	5
4. The inverse problem of the calculus of variations	6
5. Some important topics not covered in these lectures	7
Chapter 2. External and generalized symmetries	9
1. Jet bundles	9
2. Systems of differential equations	12
3. External symmetries	13
4. Classical symmetry reduction	15
5. Contact transformations and Bäcklund's Theorem	17
6. Generalized symmetries of differential equations	19
7. Generalized symmetries and conservation laws	22
Chapter 3. Internal, external and generalized symmetries	27
1. Internal symmetries	27
2. Normal systems of ordinary differential equations	28
3. Under-determined systems of ordinary differential equations	29
4. Contact conditions for ordinary differential equations	32
5. Contact conditions for partial differential equations	35
Chapter 4. Transformations of surfaces	39
1. The method of Laplace	39
2. The Laplace transformation for surfaces	42
3. An application to the $\mathfrak{sl}_2$ Toda field theory	44
Chapter 5. Transformations of submanifolds	47
1. A multi-dimensional geometric Laplace transformation	47
2. Transformations of systems of partial differential equations	51
Chapter 6. Hamiltonian systems of conservation laws	57
1. Systems of conservation laws and their local geometry	57
2. Strongly hyperbolic systems rich in conservation laws	60
3. The Laplace transformation for strongly hyperbolic systems	63
Chapter 7. The variational bi-complex	67

1. Jet bundles of infinite order	67
2. The tautological variational bi-complex on $J^\infty(E)$	69
Chapter 8. The inverse problem of the calculus of variations	75
1. The local, global and equivariant inverse problems	75
2. Edge complexes and obstructions	78
3. Variational principles and symmetry reduction	81
Chapter 9. Conservation laws and Darboux integrability	83
1. Scalar hyperbolic partial differential equations in the plane	83
2. The constrained variational bi-complex	85
3. Conservation laws	86
4. The generalized Laplace invariants	89
5. Vanishing theorems for conservation laws	92
6. Darboux integrability	93
7. Further applications	97
Chapter 10. Characteristic cohomology of differential systems	99
1. Exterior differential systems	99
2. Some existence theorems for integral manifolds	101
3. Involutive systems and the Cartan-Kähler theorem	103
4. Characteristic cohomology	108
Bibliography	111