

Contents

Preface *page viii*

Part 1. Multiplication on the tangent bundle

1	Introduction to part 1	3
1.1	First examples	4
1.2	Fast track through the results	5
2	Definition and first properties of F-manifolds	9
2.1	Finite-dimensional algebras	9
2.2	Vector bundles with multiplication	11
2.3	Definition of F-manifolds	14
2.4	Decomposition of F-manifolds and examples	16
2.5	F-manifolds and potentiality	19
3	Massive F-manifolds and Lagrange maps	23
3.1	Lagrange property of massive F-manifolds	23
3.2	Existence of Euler fields	26
3.3	Lyashko–Looijenga maps and graphs of Lagrange maps	29
3.4	Miniversal Lagrange maps and F-manifolds	32
3.5	Lyashko–Looijenga map of an F-manifold	35
4	Discriminants and modality of F-manifolds	40
4.1	Discriminant of an F-manifold	40
4.2	2-dimensional F-manifolds	44
4.3	Logarithmic vector fields	47
4.4	Isomorphisms and modality of germs of F-manifolds	52
4.5	Analytic spectrum embedded differently	56

5	Singularities and Coxeter groups	61
5.1	Hypersurface singularities	61
5.2	Boundary singularities	69
5.3	Coxeter groups and F-manifolds	75
5.4	Coxeter groups and Frobenius manifolds	82
5.5	3-dimensional and other F-manifolds	87
Part 2. Frobenius manifolds, Gauß–Manin connections, and moduli spaces for hypersurface singularities		
6	Introduction to part 2	99
6.1	Construction of Frobenius manifolds for singularities	100
6.2	Moduli spaces and other applications	104
7	Connections over the punctured plane	109
7.1	Flat vector bundles on the punctured plane	109
7.2	Lattices	113
7.3	Saturated lattices	116
7.4	Riemann–Hilbert–Birkhoff problem	120
7.5	Spectral numbers globally	128
8	Meromorphic connections	131
8.1	Logarithmic vector fields and differential forms	131
8.2	Logarithmic pole along a smooth divisor	134
8.3	Logarithmic pole along any divisor	139
8.4	Remarks on regular singular connections	143
9	Frobenius manifolds and second structure connections	145
9.1	Definition of Frobenius manifolds	145
9.2	Second structure connections	148
9.3	First structure connections	154
9.4	From the structure connections to metric and multiplication	157
9.5	Massive Frobenius manifolds	160
10	Gauß–Manin connections for hypersurface singularities	165
10.1	Semiuniversal unfoldings and F-manifolds	165
10.2	Cohomology bundle	167
10.3	Gauß–Manin connection	170
10.4	Higher residue pairings	179
10.5	Polarized mixed Hodge structures and opposite filtrations	183
10.6	Brieskorn lattice	188

11	Frobenius manifolds for hypersurface singularities	195
	11.1 Construction of Frobenius manifolds	195
	11.2 Deformed flat coordinates	205
	11.3 Remarks on mirror symmetry	211
	11.4 Remarks on oscillating integrals	212
12	μ -constant stratum	218
	12.1 Canonical complex structure	218
	12.2 Period map and infinitesimal Torelli	224
13	Moduli spaces for singularities	230
	13.1 Compatibilities	230
	13.2 Symmetries of singularities	235
	13.3 Global moduli spaces for singularities	240
14	Variance of the spectral numbers	248
	14.1 Socle field	248
	14.2 G-function of a massive Frobenius manifold	251
	14.3 Variance of the spectrum	256
	<i>Bibliography</i>	260
	<i>Index</i>	269