

# Table of Contents

<b>Introduction</b>	13
<b>Chapter 1 Elements of Differential Geometry</b>	33
1.1 Tangent Bundles	33
1.2 Vector Fields	35
1.3 Cotangent Bundles	39
1.4 Tensors	40
1.5 Tensors Fields	42
1.6 Exterior Product	43
1.7 Differential Forms	46
1.8 The de Rham Complex	48
1.9 The Codifferential, Hodge Star and Laplace-Beltrami Operators	49
<b>Chapter 2 Elements of Functional Analysis</b>	55
2.1 Transpose Operators	55
2.2 The Riesz Representation Theorem	56
2.3 Closed Operators	58
2.4 Compact Operators	59
2.5 The Riesz–Schauder Theory	59
2.6 Fredholm Operators	61
2.7 Adjoint Operators	62
2.8 The Hilbert–Schmidt Theory	64
2.9 Theory of Semigroups	65
<b>Chapter 3 Elements of Markov Processes</b>	69
3.1 Conditional Probabilities	69
3.2 Brownian Motion	70

3.3	Markov Processes	71
3.4	Markov Transition Functions and Feller Semigroups	73
3.5	Theory of Feller Semigroups	78
<b>Chapter 4 Elements of Partial Differential Equations</b>		85
4.1	Sobolev Spaces	85
4.2	Fourier Integral Operators	90
4.3	Pseudo-Differential Operators	96
4.4	Pseudo-Differential Operators on a Manifold	101
4.5	Elliptic Pseudo-Differential Operators and their Indices	103
4.6	Potentials and Pseudo-Differential Operators	115
4.7	Spaces of Currents	118
<b>Chapter 5 Index Formulas for the de Rham Complex</b>		121
5.1	The Boundaryless Case	121
5.2	The Bounded Case	127
<b>Chapter 6 The Hodge–Kodaira Decomposition Theorem</b>		141
<b>Chapter 7 The Exterior Derivative and the Co-differential Operator</b>		147
7.1	Elementary Formulas	147
7.2	The Operators $\bar{d}$ and $\bar{d}^*$	152
7.3	The Relative Hodge–Kodaira Decomposition Theorem	166
7.4	The Hodge–Kodaira Decomposition Theorem with Boundary Condition	173
<b>Chapter 8 The Operator <math>D</math></b>		179
<b>Chapter 9 The Long Exact Sequence and the Operator <math>D</math></b>		187
<b>Chapter 10 Proof of Theorem 9.3</b>		195
<b>Bibliography</b>		207
<b>Subject Index</b>		211