

Contents

0	Introduction. Examples and Motivation	13
0.1	What Is a Manifold with Singularities?	13
0.1.1	A circular cone	15
0.1.2	A circular cusp	16
0.1.3	Conclusions	18
0.2	The Semiclassical Approximation	19
0.2.1	The conical case	19
0.2.2	The Stokes phenomenon. Generalizations	28
0.2.3	The cuspidal case	30
0.3	Finiteness Theorems (the Fredholm Property)	38
0.3.1	Asymptotic behavior and statement of the problem	38
0.3.2	Operator algebras	41
0.4	Conclusions	47
I	Generalities	51
1	Structure Rings on Singular Manifolds	53
1.1	General Considerations	53
1.1.1	Local rings	53
1.1.2	Differential operators and Riemannian metrics	55
1.2	Power Stabilization (Cuspidal Points)	58
1.2.1	The local ring	58
1.2.2	Differential operators and Riemannian metrics	58
1.3	Exponential Stabilization (Conical Points)	60
1.3.1	The local ring	60
1.3.2	Differential operators and Riemannian metrics	61
1.4	Exponential Stabilization of Arbitrary Degree	62
1.4.1	The local ring	62
1.4.2	Differential operators and Riemannian metrics	63
1.5	Strong Exponential Stabilization	64
1.5.1	The local ring	64

1.5.2	Differential operators and Riemannian metrics	65
1.6	General Types of Singularities	66
1.6.1	Conification	66
1.6.2	Edgification	67
2	Interaction of Asymptotic Expansions	69
2.1	Examples	70
2.2	The General Statement	73
2.3	The Two-Dimensional Case	75
2.3.1	Propagation of singularities	75
2.3.2	Asymptotics near the intersection	77
2.4	The Multidimensional Case	81
2.4.1	Propagation of singularities	81
2.4.2	Asymptotics near the intersection	82
3	Resurgent Analysis of Functions of Polynomial Growth	85
3.1	The Resurgent Representation	85
3.1.1	Definition and main properties	85
3.1.2	Invertibility properties of the resurgent representation	91
3.2	Asymptotic Expansions and the Stokes Phenomenon	92
3.2.1	Resurgent functions with simple singularities	93
3.2.2	The Stokes phenomenon and the connection homomorphism	95
3.2.3	Conditions of single-valuedness	99
3.2.4	Asymptotic expansions near focal points	106
3.3	A Classification of Asymptotic Expansions of Functions of Polynomial Growth	110
3.3.1	The generalized resurgent representation	111
3.3.2	Types of asymptotic expansions	113
II	Elliptic Equations	121
4	Asymptotic Solutions on Manifolds with Conical Singularities	123
4.1	The Construction of Resurgent Solutions	124
4.1.1	Statement of the problem	124
4.1.2	Reduction to a resurgent equation	125
4.1.3	Solving the resurgent equations	126
4.2	Applications and Examples	129
4.2.1	Applications	129
4.2.2	Elliptic equations on the cone	130
4.2.3	Elliptic equations on a manifold with an edge	133

5	Asymptotic Solutions on Manifolds with Cusp-Type Singularities	135
5.1	Examples	135
5.1.1	The cusp of order 1	137
5.1.2	The cusp of order 2	140
5.2	Formal Theory	143
5.2.1	The general asymptotic expansion	144
5.2.2	Analysis of the asymptotic expansion	149
5.2.3	Explicit computation of the coefficients	151
5.3	The Construction of Resurgent Solutions	159
5.3.1	The case of a simple cusp	160
5.3.2	The case of a cusp of higher multiplicity	165
6	Asymptotic Solutions on Manifolds with Corner-Type Singularities	171
6.1	Example	171
6.2	The Construction of Resurgent Solutions	178
6.2.1	Statement of the problem	178
6.2.2	The solvability theorem	182
6.2.3	Investigation of the singularity set of the solution	184
6.2.4	Asymptotics of solutions near the vertex	186
6.2.5	The case of resurgent functions with simple singularities	191
6.3	Two-Dimensional Problems	194
6.3.1	Resurgent solutions	194
6.3.2	The solvability of an analytic family of one-dimensional problems	196
7	General Asymptotic Theory	201
7.1	Resurgent Analysis	202
7.1.1	Preliminaries	202
7.1.2	Analytic groups and integral representations	204
7.1.3	Resurgent elements of the algebra	211
7.1.4	Examples	215
7.1.5	The parametric case and the Stokes phenomenon	218
7.2	Asymptotics of Solutions	220
7.2.1	Description of the class of equations	220
7.2.2	The asymptotic expansion (the first case)	221
7.2.3	The asymptotic expansion (the second case)	222
7.2.4	A -differential equations	224
7.2.5	The solution of nonhomogeneous equations	225
7.3	Deformations of Integral Transforms and Equations	229
7.3.1	General theory	229
7.3.2	Examples	231

8	Finiteness Theorems	243
8.1	Function Spaces	243
8.1.1	Preliminaries	244
8.1.2	Resurgent representations	245
8.1.3	Scales of function spaces	248
8.2	Spaces with Asymptotics	250
8.2.1	Preliminary considerations	250
8.2.2	Main definitions	251
8.2.3	Elements with simple singularities	254
8.3	Operator Algebras	255
8.3.1	The description of generators	255
8.3.2	Functions of generators	256
8.3.3	Construction of the operator algebras	258
8.3.4	Ellipticity and regularizers	260
8.4	The Finiteness Theorem for Differential Equations on Manifolds with Cuspidal Points	262
8.4.1	Statement of the problem	262
8.4.2	The construction of local regularizers	265
8.4.3	The global regularizer and the Fredholm property	266
8.5	Asymptotic Expansions of Solutions	267
8.5.1	A preliminary transformation	268
8.5.2	A priori properties of the Borel transform of a solution	269
8.5.3	Multiplication by a function in the Borel representation	270
8.5.4	Proof of the theorem on endless continuability	274
8.6	Deformation of Resurgent Transforms	275
8.6.1	Definition of the deformation	275
8.6.2	Deformation of the operators and the index theorem	278
III	Hyperbolic Equations	281
9	Equations of Borel–Fuchs Type	283
9.1	Solution of the Problem “in the Small”	284
9.1.1	Degeneration of the first degree	285
9.1.2	Degeneration of higher degree	290
9.2	Solution of the Problem “in the Large”	296
9.2.1	Degeneracy of the first degree	296
9.2.2	Degeneracy of higher degree	300
9.3	Nonstationary problems in abstract algebras	302
9.3.1	General theory	302
9.3.2	Example	308

10 Vibration of Elastic Shells with Conical Points **313**

- 10.1 The Model Example 314
 - 10.1.1 Reduction of the problem 315
 - 10.1.2 Asymptotic expansions of solutions 316
- 10.2 The General Case 330
 - 10.2.1 The case of higher multiplicities 330
 - 10.2.2 The multidimensional case 332

Appendices **341**

A $\partial/\partial s$ -Transform of Ramified Analytic Functions **343**

- A.1 Definition and Main Properties 344
 - A.1.1 Auxiliary statements: Feynman integrals 344
 - A.1.2 The Thom theorem 348
 - A.1.3 Definition of the transform 351
 - A.1.4 Commutation formulas 354
- A.2 Functions with Simple Singularities 355
 - A.2.1 Definitions 355
 - A.2.2 The main theorem 356
 - A.2.3 Proof of the main theorem 356
 - A.2.4 Calculation of the leading term 359

B Some Elements of Noncommutative Analysis **361**

- B.1 Formal Arithmetics 362
 - B.1.1 The index permutation formula 363
 - B.1.2 The commutation formula 364
 - B.1.3 The derivation formula 364
 - B.1.4 Higher-order expansions 364
- B.2 The Method of Ordered Representations 365

Bibliography **367**

Index **373**