

Contents

<i>Introduction</i>	<i>ix</i>
1 Coxeter Groups	1
1.1 Words, lengths, presentations of groups	2
1.2 Coxeter groups, systems, diagrams	2
1.3 Reflections, roots	3
1.4 Roots and the length function	6
1.5 More on roots and lengths	9
1.6 Generalized reflections	11
1.7 Exchange, Deletion conditions	12
1.8 The Bruhat order	16
1.9 Special subgroups of Coxeter groups	20
2 Seven Infinite Families	25
2.1 Three spherical families	26
2.2 Four affine families	27
3 Chamber Complexes	31
3.1 Chamber complexes	32
3.2 The uniqueness lemma	35
3.3 Foldings, walls, reflections	36
3.4 Coxeter complexes	40
3.5 Characterization by foldings and walls	43
3.6 Corollaries on foldings	48
4 Buildings	51
4.1 Apartments and buildings: definitions	52
4.2 Canonical retractions to apartments	53
4.3 Apartments are Coxeter complexes	54
4.4 Labels, links	56
4.5 Convexity of apartments	59
4.6 Spherical buildings	59
5 BN-pairs from Buildings	63
5.1 BN-pairs: definitions	64
5.2 BN-pairs from buildings	64
5.3 Parabolic (special) subgroups	70

5.4	Further Bruhat-Tits decompositions	71
5.5	Generalized BN-pairs	73
5.6	The spherical case	75
5.7	Buildings from BN-pairs	79
6	Generic and Hecke Algebras	87
6.1	Generic algebras	88
6.2	Iwahori-Hecke algebras	92
6.3	Generalized Iwahori-Hecke algebras	95
7	Geometric Algebra	101
7.1	$GL(n)$ (a prototype)	102
7.2	Bilinear and hermitian forms	106
7.3	Extending isometries	111
7.4	Parabolics	114
8	Examples in Coordinates	119
8.1	Symplectic groups	120
8.2	Orthogonal groups $O(n,n)$	123
8.3	Orthogonal groups $O(p,q)$	125
8.4	Unitary groups in coordinates	127
9	Spherical Construction for $GL(n)$	131
9.1	Construction	132
9.2	Verification of the building axioms	132
9.3	Action of $GL(n)$ on the building	136
9.4	The spherical BN-pair in $GL(n)$	137
9.5	Analogous treatment of $SL(n)$	139
9.6	Symmetric groups as Coxeter groups	140
10	Spherical Construction for Isometry Groups	143
10.1	Constructions	144
10.2	Verification of the building axioms	145
10.3	The action of the isometry group	150
10.4	The spherical BN-pair	151
10.5	Analogues for similitude groups	154
11	Spherical Oriflamme Complex	157
11.1	Oriflamme construction for $SO(n,n)$	158
11.2	Verification of the building axioms	159
11.3	The action of $SO(n,n)$	164
11.4	The spherical BN-pair in $SO(n,n)$	168
11.5	Analogues for $GO(n,n)$	170

12 Reflections, Root Systems and Weyl Groups	173
12.1 Hyperplanes, chambers, walls	174
12.2 Reflection groups are Coxeter groups	177
12.3 Finite reflection groups	181
12.4 Affine reflection groups	186
12.5 Affine Weyl groups	190
13 Affine Coxeter Complexes	197
13.1 Tits' cone model of Coxeter complexes	198
13.2 Positive-definite (spherical) case	202
13.3 A lemma from Perron-Frobenius	203
13.4 Local finiteness of Tits' cones	205
13.5 Definition of geometric realizations	207
13.6 Criterion for affineness	209
13.7 The canonical metric	214
13.8 The seven infinite families	216
14 Affine Buildings	221
14.1 Affine buildings, trees: definitions	222
14.2 Canonical metrics on affine buildings	222
14.3 Negative curvature inequality	225
14.4 Contractibility	227
14.5 Completeness	228
14.6 Bruhat-Tits fixed-point theorem	229
14.7 Maximal compact subgroups	230
14.8 Special vertices, compact subgroups	235
15 Combinatorial Geometry	239
15.1 Minimal and reduced galleries	240
15.2 Characterizing apartments	241
15.3 Existence of prescribed galleries	242
15.4 Configurations of three chambers	244
15.5 Subsets of apartments	247
16 Spherical Building at Infinity	253
16.1 Sectors	254
16.2 Bounded subsets of apartments	255
16.3 Lemmas on isometries	256
16.4 Subsets of apartments	260
16.5 Configurations of chamber and sector	265
16.6 Sector and three chambers	267
16.7 Configurations of two sectors	270
16.8 Geodesic rays	274

16.9	The spherical building at infinity	277
16.10	Induced maps at infinity	284
17	Applications to Groups	289
17.1	Induced group actions at infinity	290
17.2	BN-pairs, parahorics and parabolics	291
17.3	Translations and Levi components	293
17.4	Levi filtration by sectors	294
17.5	Bruhat and Cartan decompositions	297
17.6	Iwasawa decomposition	297
17.7	Maximally strong transitivity	299
17.8	Canonical translations	301
18	Lattices, p-adic Numbers, Discrete Valuations	305
18.1	p-adic numbers	306
18.2	Discrete valuations	309
18.3	Hensel's Lemma	311
18.4	Lattices	313
18.5	Some topology	314
18.6	Iwahori decomposition for $GL(n,k)$	317
19	Affine Building for $SL(n)$	321
19.1	Construction	322
19.2	Verification of the building axioms	324
19.3	The action of $SL(V)$	329
19.4	The Iwahori subgroup 'B'	330
19.5	The maximal apartment system	332
20	Affine Buildings for Isometry Groups	335
20.1	Affine buildings for alternating spaces	336
20.2	The double oriflamme complex	338
20.3	The (affine) single oriflamme complex	340
20.4	Verification of the building axioms	344
20.5	Group actions on the buildings	347
20.6	Iwahori subgroups	349
20.7	The maximal apartment systems	351
	<i>Bibliography</i>	353
	<i>Index</i>	371