
Contents

Preface	ix
1 General theory	1
1.1 History	1
1.2 Actions and G -spaces	2
1.3 Orbits and transitivity	3
1.4 Transitive groups and coset spaces	4
1.5 Sylow's Theorem	6
1.6 Regular groups	7
1.7 Groups with regular normal subgroups	9
1.8 Multiple transitivity	10
1.9 Primitivity	11
1.10 Wreath products	11
1.11 Orbitals	13
1.12 Sharp k -transitivity	15
1.13 The Schreier–Sims algorithm	17
1.14 Jerrum's filter	19
1.15 The length of S_n	20
1.16 At the keyboard	21
1.17 Appendix: Cycles and parity	25
1.18 Exercises	27
2 Representation theory	35
2.1 Historical note	35
2.2 Centraliser algebra	36
2.3 The Orbit-Counting Lemma	37
2.4 Character theory	40

2.5	The permutation character	42
2.6	The diagonal group	44
2.7	Frobenius–Schur index	45
2.8	Parker’s Lemma	47
2.9	Characters of abelian groups	51
2.10	Characters of the symmetric group	53
2.11	Appendix: Möbius inversion	56
2.12	Exercises	57
3	Coherent configurations	63
3.1	Introduction	63
3.2	Algebraic theory	66
3.3	Association schemes	68
3.4	Algebra of association schemes	73
3.5	Example: Strongly regular graphs	76
3.6	The Hoffman–Singleton graph	79
3.7	Automorphisms	83
3.8	Valency bounds	85
3.9	Distance-transitive graphs	87
3.10	Multiplicity bounds	90
3.11	Duality	92
3.12	Wielandt’s Theorem	94
3.13	Exercises	95
4	The O’Nan–Scott Theorem	99
4.1	Introduction	99
4.2	Precursors	100
4.3	Product action and basic groups	102
4.4	Some basic groups	104
4.5	The O’Nan–Scott Theorem	105
4.6	Maximal subgroups of S_n	107
4.7	The finite simple groups	108
4.8	Application: Multiply-transitive groups	110
4.9	Application: Degrees of primitive groups	111
4.10	Application: Orders of primitive groups	112
4.11	Application: The length of S_n	117
4.12	Application: Distance-transitive graphs	118
4.13	Bases	120
4.14	Geometric groups and IBIS groups	123
4.15	Appendix: Matroids	125
4.16	Exercises	127

5 Oligomorphic groups	131
5.1 The random graph	131
5.2 Oligomorphic groups	135
5.3 First-order logic	135
5.4 Automorphism groups and topology	138
5.5 Countably categorical structures	139
5.6 Homogeneous structures	141
5.7 Cycle index	143
5.8 A graded algebra	145
5.9 Monotonicity	147
5.10 Set-transitive groups	148
5.11 Growth rates	150
5.12 On complementation and switching	153
5.13 Appendix: Cycle index	157
5.14 Exercises	160
6 Miscellanea	165
6.1 Finitary permutation groups	165
6.2 Neumann's Lemma	167
6.3 Cofinitary permutation groups	168
6.4 Theorems of Blichfeldt and Maillet	170
6.5 Cycle-closed permutation groups	173
6.6 Fixed-point-free elements	173
6.7 The Orbit-Counting Lemma revisited	176
6.8 Jordan groups	179
6.9 Orbits on moieties	182
6.10 Exercises	183
7 Tables	187
7.1 Simple groups of Lie type	188
7.2 Sporadic simple groups	192
7.3 Affine 2-transitive groups	194
7.4 Almost simple 2-transitive groups	196
7.5 Exercises	198
Bibliography	199
Index	213