

Contents

Preface	xi
Summary of Global Notation	xv
Part I - Prerequisites	1
Chapter 1 Prerequisites from Category Theory	3
1 Definitions	3
2 Universal Constructions	4
3 Basic Notions from Categorical Theory	5
Chapter 2 Prerequisites from Point Set Topology	7
1 Definitions and Elementary Constructions	7
2 Separation	8
3 Compactness	9
4 Connectedness	10
5 Coverings and Paracompactness	10
6 Coherent and Compactly Generated Topologies	11
7 CW -complexes	12
Chapter 3 The Fundamental Group	15
1 Homotopy	15
2 Covering Spaces	17
3 Van Kampen's Theorem	20
Chapter 4 Homological Algebra	21
1 Chain Complexes	21
2 Direct Limits	24
3 Derived Functors	25
4 Tensor Products, Duals, and Universal Coefficient Theorems	28
5 Adjoint Functors	30
6 Relative Derived Functors	30
7 Derived Functors of \lim	31
Chapter 5 Homology of Spaces	33
1 Eilenberg-Steenrod Homology Axioms	33
2 Singular Homology Theory	34
3 Homology Calculations and Elementary Applications	38
4 Homology of CW -complexes; Cellular Homology	39
5 Acyclic Models	41
6 Singular Cohomology	43
7 Relative Cohomology	44

Chapter 6	Manifolds	47
1	Definitions	47
2	Orientation on Manifolds	47
3	Poincaré Duality	48
Part II - Homotopy Theory		51
Chapter 7	Higher Homotopy Theory	53
1	Fibrations and Cofibrations	53
2	Group Structures on Sets of Homotopy Classes	62
3	Homology of a Loop-suspension	66
4	Relation between Homotopy Groups and Homology Groups	67
5	Reduction to the Case of Simply Connected CW -Complexes	72
6	Diagrams of Exact Homotopy Sequences	76
7	Ganea's Theorem	81
8	Whitehead and Samelson Products	83
9	James Construction	84
10	Lusternik-Schnirrelmann Category	88
Chapter 8	Simplicial Sets	89
1	Definitions and First Principles	89
2	Simplicial Fibrations and Kan Complexes	92
3	Simplicial Groups	94
4	The Singular Complex	94
5	Simplicial Abelian Groups	95
6	Simplicial Approximation	97
Chapter 9	Fibre Bundles and Classifying Spaces	99
1	Fibre Bundles	99
2	Classifying Principal Bundles and Milnor's Construction	100
Chapter 10	Hopf Algebras and Graded Lie Algebras	105
1	Algebras and Coalgebras	105
2	Tensor Products and Cotensor Products	107
3	Kernels and Cokernels	107
4	Primitives and Indecomposables	109
5	Graded Lie Algebras	109
6	The Canonical Conjugation	110
Chapter 11	Spectral Sequences	113
1	Filtrations	113
2	Definition of Spectral Sequence	114
3	Exact Couples	116
4	Exact Couple of a Filtered Object	119
5	Bigraded Spectral Sequences	121
6	Multiplicative Spectral Sequences	122

7	Spectral Sequence of a Double Complex	123
8	Bockstein Spectral Sequence	125
9	Serre Spectral Sequence	126
10	Adams-Hilton Models	136
11	Eilenberg-Moore Spectral Sequence	141
12	Grothendieck Spectral Sequence	145
Chapter 12	Localization and Completion	147
1	Triples and Cotriples	147
2	The Bousfield-Kan Construction	147
Chapter 13	Generalized Homology and Stable Homotopy	153
1	Generalized Homology	153
2	Stable Homotopy Groups	156
3	Spectra	156
4	Eilenberg-MacLane Spaces and Brown Representability	158
Chapter 14	Cohomology Operations and the Steenrod Algebra	161
1	Primary Cohomology Operations	161
2	The Steenrod Algebra as a Hopf Algebra	162
3	Construction of the Steenrod Reduced Power Operations	163
4	Calculation of the Steenrod Algebra	170
5	Dual of the Steenrod Algebra	174
6	Sample Applications of the Steenrod Algebra	175
7	Secondary Cohomology Operations	179
Bibliography		181
Index		183