Contents

1 Differentiable Manifolds 1
§1–1 Definition of Differentiable Manifolds 1
§1–2 Tangent Spaces 9
§1–3 Submanifolds 18
§1–4 Frobenius' Theorem 29

2 Multilinear Algebra 39
§2–1 Tensor Products 39
§2–2 Tensors 47
§2–3 Exterior Algebra 52

3 Exterior Differential Calculus 65
§3–1 Tensor Bundles and Vector Bundles 65
§3–2 Exterior Differentiation 74
§3–3 Integrals of Differential Forms 85
§3–4 Stokes' Formula 92

4 Connections 101
§4–1 Connections on Vector Bundles 101
§4–2 Affine Connections 113
§4–3 Connections on Frame Bundles 121

5 Riemannian Geometry 133
§5–1 The Fundamental Theorem of Riemannian Geometry 133
§5–2 Geodesic Normal Coordinates 143
§5–3 Sectional Curvature 155
§5–4 The Gauss–Bonnet Theorem 162

6 Lie Groups and Moving Frames 173
§6–1 Lie Groups 173
§6–2 Lie Transformation Groups 186
§6–3 The Method of Moving Frames 198

§6–4 Theory of Surfaces 210

7 Complex Manifolds **221**
§7–1 Complex Manifolds 221
§7–2 The Complex Structure on a Vector Space 227
§7–3 Almost Complex Manifolds 236
§7–4 Connections on Complex Vector Bundles 244
§7–5 Hermitian Manifolds and Kählerian Manifolds 256

8 Finsler Geometry **265**
§8–1 Preliminaries 265
§8–2 Geometry on the Projectivised Tangent Bundle (PTM) and the Hilbert Form 267
§8–3 The Chern Connection 273
§8–3.1 Determination of the Connection 274
§8–3.2 The Cartan Tensor and Characterization of Riemannian Geometry 280
§8–3.3 Explicit Formulas for the Connection Forms in Natural Coordinates 283
§8–4 Structure Equations and the Flag Curvature 288
§8–4.1 The Curvature Tensor 289
§8–4.2 The Flag Curvature and the Ricci Curvature 293
§8–4.3 Special Finsler Spaces 295
§8–5 The First Variation of Arc Length and Geodesics 297
§8–6 The Second Variation of Arc Length and Jacobi Fields 306
§8–7 Completeness and the Hopf-Rinow Theorem 314
§8–8 The Theorems of Bonnet-Myers and Synge 325

A Historical Notes **331**
§A–1 Classical Differential Geometry 331
§A–2 Riemannian Geometry 331
§A–3 Manifolds 332
§A–4 Global Geometry 332

B Differential Geometry and Theoretical Physics **335**
§B–1 Dynamics and Moving Frames 336
§B–2 Theory of Surfaces, Solitons and the Sigma Model 338
§B–3 Gauge Field Theory 340
§B–4 Conclusion 341

References **343**

Index **347**