Contents

Introduction xiii

Chapter 1. Preliminaries 1

§1.1. Bundles, connections and characteristic classes 1

1.1.1. Vector bundles and connections 1

1.1.2. Chern-Weil theory 12

§1.2. Basic facts about elliptic equations 16

§1.3. Clifford algebras and Dirac operators 27

1.3.1. Clifford algebras and their representations 27

1.3.2. The *Spin* and $Spin^c$ groups 34

1.3.3. *Spin* and $spin^c$ structures 40

1.3.4. Dirac operators associated to *spin* and $spin^c$ structures 46

§1.4. Complex differential geometry 53

1.4.1. Elementary complex differential geometry 54

1.4.2. Cauchy-Riemann operators 68

1.4.3. Dirac operators on almost Kähler manifolds 80

§1.5. Fredholm theory 85

1.5.1. Continuous families of elliptic operators 85

1.5.2. Genericity results 100

Chapter 2. The Seiberg-Witten Invariants 103

§2.1. Seiberg-Witten monopoles 103

2.1.1. The Seiberg-Witten equations 103
2.1.2. The functional set-up 108

§2.2. The structure of the Seiberg-Witten moduli spaces 114
2.2.1. The topology of the moduli spaces 114
2.2.2. The local structure of the moduli spaces 119
2.2.3. Generic smoothness 131
2.2.4. Orientability 137

§2.3. The structure of the Seiberg-Witten invariants 139
2.3.1. The universal line bundle 139
2.3.2. The case $b_2^+ > 1$ 142
2.3.3. The case $b_2^+ = 1$ 154
2.3.4. Some examples 169

§2.4. Applications 176
2.4.1. The Seiberg-Witten equations on cylinders 176
2.4.2. The Thom conjecture 183
2.4.3. Negative definite smooth 4-manifolds 188

Chapter 3. Seiberg-Witten Equations on Complex Surfaces 195

§3.1. A short trip in complex geometry 195
3.1.1. Basic notions 195
3.1.2. Examples of complex surfaces 213
3.1.3. Kodaira classification of complex surfaces 225

§3.2. Seiberg-Witten invariants of Kähler surfaces 228
3.2.1. Seiberg-Witten equations on Kähler surfaces 228
3.2.2. Monopoles, vortices and divisors 233
3.2.3. Deformation theory 242

§3.3. Applications 245
3.3.1. A nonvanishing result 246
3.3.2. Seiberg-Witten invariants of simply connected elliptic surfaces 252
3.3.3. The failure of the h-cobordism theorem in four dimensions 274
3.3.4. Seiberg-Witten equations on symplectic 4-manifolds 275

Chapter 4. Gluing Techniques 283

§4.1. Elliptic equations on manifolds with cylindrical ends 283
4.1.1. Manifolds with cylindrical ends 283
4.1.2. The Atiyah-Patodi-Singer index theorem 286
4.1.3. Eta invariants and spectral flows 292
4.1.4. The Lockhart-McOwen theory 298
4.1.5. Abstract linear gluing results 303
4.1.6. Examples 309
§4.2. Finite energy monopoles 328
4.2.1. Regularity 328
4.2.2. Three-dimensional monopoles 330
4.2.3. Asymptotic behavior. Part I 338
4.2.4. Asymptotic behavior. Part II 344
4.2.5. Proofs of some technical results 359
§4.3. Moduli spaces of finite energy monopoles: Local aspects 368
4.3.1. Functional set-up 368
4.3.2. The Kuranishi picture 383
4.3.3. Virtual dimensions 397
4.3.4. Reducible finite energy monopoles 404
§4.4. Moduli spaces of finite energy monopoles: Global aspects 409
4.4.1. Genericity results 409
4.4.2. Compactness properties 412
4.4.3. Orientability issues 425
§4.5. Cutting and pasting of monopoles 428
4.5.1. Some basic gluing constructions 429
4.5.2. Gluing monopoles: Local theory 435
4.5.3. The local surjectivity of the gluing construction 447
4.5.4. Gluing monopoles: Global theory 451
§4.6. Applications 460
4.6.1. Vanishing results 461
4.6.2. Blow-up formula 465

Epilogue 471

Bibliography 473

Index 481