

Contents

Preface xi
A Note to the Reader xv

Part I GENERAL TOPOLOGY

Chapter 1 Set Theory and Logic 3
 1 Fundamental Concepts 4
 2 Functions 15
 3 Relations 21
 4 The Integers and the Real Numbers 30
 5 Cartesian Products 36
 6 Finite Sets 39
 7 Countable and Uncountable Sets 44
 *8 The Principle of Recursive Definition 52
 9 Infinite Sets and the Axiom of Choice 57
 10 Well-Ordered Sets 62
 *11 The Maximum Principle 68
 *Supplementary Exercises: Well-Ordering 72

Chapter 2 Topological Spaces and Continuous Functions	75
12 Topological Spaces	75
13 Basis for a Topology	78
14 The Order Topology	84
15 The Product Topology on $X \times Y$	86
16 The Subspace Topology	88
17 Closed Sets and Limit Points	92
18 Continuous Functions	102
19 The Product Topology	112
20 The Metric Topology	119
21 The Metric Topology (continued)	129
*22 The Quotient Topology	136
*Supplementary Exercises: Topological Groups	145
Chapter 3 Connectedness and Compactness	147
23 Connected Spaces	148
24 Connected Subspaces of the Real Line	153
*25 Components and Local Connectedness	159
26 Compact Spaces	163
27 Compact Subspaces of the Real Line	172
28 Limit Point Compactness	178
29 Local Compactness	182
*Supplementary Exercises: Nets	187
Chapter 4 Countability and Separation Axioms	189
30 The Countability Axioms	190
31 The Separation Axioms	195
32 Normal Spaces	200
33 The Urysohn Lemma	207
34 The Urysohn Metrization Theorem	214
*35 The Tietze Extension Theorem	219
*36 Imbeddings of Manifolds	224
*Supplementary Exercises: Review of the Basics	228
Chapter 5 The Tychonoff Theorem	230
37 The Tychonoff Theorem	230
38 The Stone-Čech Compactification	237
Chapter 6 Metrization Theorems and Paracompactness	243
39 Local Finiteness	244
40 The Nagata-Smirnov Metrization Theorem	248
41 Paracompactness	252
42 The Smirnov Metrization Theorem	261

Chapter 7 Complete Metric Spaces and Function Spaces	263
43 Complete Metric Spaces	264
*44 A Space-Filling Curve	271
45 Compactness in Metric Spaces	275
46 Pointwise and Compact Convergence	281
47 Ascoli's Theorem	290
Chapter 8 Baire Spaces and Dimension Theory	294
48 Baire Spaces	295
*49 A Nowhere-Differentiable Function	300
50 Introduction to Dimension Theory	304
*Supplementary Exercises: Locally Euclidean Spaces	316

Part II ALGEBRAIC TOPOLOGY

Chapter 9 The Fundamental Group	321
51 Homotopy of Paths	322
52 The Fundamental Group	330
53 Covering Spaces	335
54 The Fundamental Group of the Circle	341
55 Retractions and Fixed Points	348
*56 The Fundamental Theorem of Algebra	353
*57 The Borsuk-Ulam Theorem	356
58 Deformation Retracts and Homotopy Type	359
59 The Fundamental Group of S^n	368
60 Fundamental Groups of Some Surfaces	370
Chapter 10 Separation Theorems in the Plane	376
61 The Jordan Separation Theorem	376
*62 Invariance of Domain	381
63 The Jordan Curve Theorem	385
64 Imbedding Graphs in the Plane	394
65 The Winding Number of a Simple Closed Curve	398
66 The Cauchy Integral Formula	403
Chapter 11 The Seifert-van Kampen Theorem	407
67 Direct Sums of Abelian Groups	407
68 Free Products of Groups	412
69 Free Groups	421
70 The Seifert-van Kampen Theorem	426
71 The Fundamental Group of a Wedge of Circles	434
72 Adjoining a Two-cell	438
73 The Fundamental Groups of the Torus and the Dunce Cap	442

Chapter 12	Classification of Surfaces	446
74	Fundamental Groups of Surfaces	446
75	Homology of Surfaces	454
76	Cutting and Pasting	457
77	The Classification Theorem	462
78	Constructing Compact Surfaces	471
Chapter 13	Classification of Covering Spaces	477
79	Equivalence of Covering Spaces	478
80	The Universal Covering Space	484
*81	Covering Transformations	487
82	Existence of Covering Spaces	494
*	Supplementary Exercises: Topological Properties and π_1	499
Chapter 14	Applications to Group Theory	501
83	Covering Spaces of a Graph	501
84	The Fundamental Group of a Graph	506
85	Subgroups of Free Groups	513
Bibliography		517
Index		519