CONTENTS

CHAPTER		PAGE
I.	RATIONALS AND IRRATIONALS	
	1. The preponderance of irrationals	1
	2. Countability	4
	3. Dense sets	5
	4. Decimal expansions	6
II.	SIMPLE IRRATIONALITIES	
	1. Introduction	15
	2. The trigonometric functions and π	16
	3. The hyperbolic, exponential, and logarithmic functions.	22
III.	CERTAIN ALGEBRAIC NUMBERS	
	1. Introduction	28
	2. Further background material	30
	3. The factorization of $x^n - 1$	33
	4. Certain trigonometric values	36
	5. Extension to the tangent	38
IV.	THE APPROXIMATION OF IRRATIONALS BY RATIONALS	
	1. The problem	42
	2. A generalization	44
	3. Linearly dependent sets	48
V.	CONTINUED FRACTIONS	
	1. The Euclidean algorithm	51
	2. Uniqueness	53
	3. Infinite continued fractions	54
	4. Infinite continued fraction expansions	59
	5. The convergents as approximations	61
	6. Periodic continued fractions	63
VI.	FURTHER DIOPHANTINE APPROXIMATIONS	
	1. A basic result	68
	2. Best possible approximations	70

CHAPTER PAGE

3. Uniform distributions ... 71
4. A proof by Fourier analysis ... 75

VII. Algebraic and Transcendental Numbers
1. Closure properties of algebraic numbers ... 83
2. A property of algebraic integers ... 85
3. Transcendental numbers ... 87
4. The order of approximation ... 88

VIII. Normal Numbers
1. Definition of a normal number ... 94
2. The measure of the set of normal numbers ... 98
3. Equivalent definitions ... 104
4. A normal number exhibited ... 112

IX. The Generalized Lindemann Theorem
1. Statement of the theorem ... 117
2. Preliminaries ... 118
3. Proof of the theorem ... 124
4. Applications of the theorem ... 131
5. Squaring the circle ... 132

X. The Gelfond–Schneider Theorem
1. Hilbert's seventh problem ... 134
2. Background material ... 135
3. Two lemmas ... 137
4. Proof of the Gelfond–Schneider theorem ... 142

List of Notation ... 151
Glossary ... 153
Reference Books ... 157
Index of Topics ... 159
Index of Names ... 163