Contents

Foreword Introduction		vii
		xv
Cha	pter 1. Preliminaries to Complex Analysis	1
1	Complex numbers and the complex plane	1
	1.1 Basic properties	1
	1.2 Convergence	5
	1.3 Sets in the complex plane	5
2	Functions on the complex plane	8
	2.1 Continuous functions	8
	2.2 Holomorphic functions	8
	2.3 Power series	14
3	Integration along curves	18
4	Exercises	24
Cha	pter 2. Cauchy's Theorem and Its Applications	32
1	Goursat's theorem	34
2	Local existence of primitives and Cauchy's theorem in a disc	37
3	Evaluation of some integrals	41
4	Cauchy's integral formulas	45
5	Further applications	53
_	5.1 Morera's theorem	53
	5.2 Sequences of holomorphic functions	53
	5.3 Holomorphic functions defined in terms of integrals	55
	5.4 Schwarz reflection principle	57
	5.5 Runge's approximation theorem	60
6	Exercises	64
7	Problems	67
Cha	pter 3. Meromorphic Functions and the Logarithm	71
1	Zeros and poles	72
2	The residue formula	76
-	2.1 Examples	77
3	Singularities and meromorphic functions	83
4	The argument principle and applications	89
-		

••	
XII	CONTENTS

5	Homotopies and simply connected domains	93
6	The complex logarithm	97
7	Fourier series and harmonic functions	101
8	Exercises	103
9	Problems	108
Cha	pter 4. The Fourier Transform	111
1	The class $\mathfrak F$	113
2	Action of the Fourier transform on $\mathfrak F$	114
3	Paley-Wiener theorem	121
4	Exercises	127
5	Problems	131
Cha	apter 5. Entire Functions	134
1	Jensen's formula	135
2	Functions of finite order	138
3	Infinite products	140
	3.1 Generalities	140
	3.2 Example: the product formula for the sine function	142
4	Weierstrass infinite products	145
5	Hadamard's factorization theorem	147
6	Exercises	153
7	Problems	156
Cha	pter 6. The Gamma and Zeta Functions	159
1	The gamma function	160
	1.1 Analytic continuation	161
	1.2 Further properties of Γ	163
2	The zeta function	168
	2.1 Functional equation and analytic continuation	168
3	Exercises	174
4	Problems	179
Cha	apter 7. The Zeta Function and Prime Number The-	
C	orem	181
1	Zeros of the zeta function	182
	1.1 Estimates for $1/\zeta(s)$	187
2	Reduction to the functions ψ and ψ_1	188
	2.1 Proof of the asymptotics for ψ_1	194
N	ote on interchanging double sums	197
3	Exercises	199

CON'	TENTS	xiii
4	Problems	203
Cha	pter 8. Conformal Mappings	205
1	Conformal equivalence and examples	206
	1.1 The disc and upper half-plane	208
	1.2 Further examples	209
	1.3 The Dirichlet problem in a strip	212
2	The Schwarz lemma; automorphisms of the disc and upper	
	half-plane	218
	2.1 Automorphisms of the disc	219
	2.2 Automorphisms of the upper half-plane	221
3	The Riemann mapping theorem	224
	3.1 Necessary conditions and statement of the theorem	224
	3.2 Montel's theorem	225
	3.3 Proof of the Riemann mapping theorem	228
4	Conformal mappings onto polygons	231
	4.1 Some examples	231
	4.2 The Schwarz-Christoffel integral	235
	4.3 Boundary behavior	238
	4.4 The mapping formula	241
	4.5 Return to elliptic integrals	245
5	Exercises	248
6	Problems	254
Cha	pter 9. An Introduction to Elliptic Functions	261
1	Elliptic functions	262
	1.1 Liouville's theorems	264
	1.2 The Weierstrass \wp function	266
2	The modular character of elliptic functions and Eisenstein	
	series	273
	2.1 Eisenstein series	273
	2.2 Eisenstein series and divisor functions	276
3	Exercises	278
4	Problems	281
Cha	pter 10. Applications of Theta Functions	283
1	Product formula for the Jacobi theta function	284
-	1.1 Further transformation laws	289
2	Generating functions	293
3	The theorems about sums of squares	296
•	3.1 The two-squares theorem	297

•	
XIV	CONTENTS

	3.2 The four-squares theorem	304
4	Exercises	309
5	Problems	314
App	pendix A: Asymptotics	318
1	Bessel functions	319
2	Laplace's method; Stirling's formula	323
3		328
4	The partition function	334
5	Problems	341
App	pendix B: Simple Connectivity and Jordan Curve	
	Theorem	344
1	Equivalent formulations of simple connectivity	345
2	The Jordan curve theorem	351
	2.1 Proof of a general form of Cauchy's theorem	361
Not	Notes and References	
Bib	Bibliography	
Symbol Glossary		373
Index		375