

Contents

Preface	xii
---------	-----

Part 1. Analysis and Geometry on Foliated Spaces

Foreword to Part 1	3
Chapter 1. The C^* -Algebra of a Foliated Space	5
§1.1. Twisted Forms and Densities	6
§1.2. Functions on Non-Hausdorff Spaces	8
§1.3. The Graph of a Foliated Space	11
§1.4. The C^* -algebra of a Foliated Space	18
§1.5. The Basic Examples	27
§1.6. Quasi-invariant Currents	37
§1.7. Representations of the Foliation C^* -algebra	48
§1.8. Minimal Foliations and their C^* -algebras	54
Chapter 2. Harmonic Measures for Foliated Spaces	61
§2.1. Existence of Harmonic Measures	62
§2.2. The Diffusion Semigroup	68
§2.3. The Markov Process	80
§2.4. Characterizations of Harmonic Measures	86
§2.5. The Ergodic Theorem	96
§2.6. Ergodic Decomposition of Harmonic Measures	99
§2.7. Recurrence	112

Chapter 3. Generic Leaves	119
§3.1. The Main Results and Examples	119
§3.2. The Holonomy Graph	122
§3.3. Proof of the Theorems	128
§3.4. Generic Geometry of Leaves	131
Part 2. Characteristic Classes and Foliations	
Foreword to Part 2	139
Chapter 4. The Euler Class of Circle Bundles	141
§4.1. Generalities about Bundles	142
§4.2. Cell Complexes	144
§4.3. The First Obstruction	148
§4.4. The Euler Class	155
§4.5. Foliated Circle Bundles	164
§4.6. Further Developments	174
Chapter 5. The Chern-Weil Construction	177
§5.1. The Chern-Weil Homomorphism	178
§5.2. The Structure of $I_n^*(\mathbb{K})$	181
§5.3. Chern Classes and Pontryagin Classes	184
Chapter 6. Characteristic Classes and Integrability	187
§6.1. The Bott Vanishing Theorem	187
§6.2. The Godbillon-Vey Class in Arbitrary Codimension	192
§6.3. Construction of the Exotic Classes	194
§6.4. Haefliger Structures and Classifying Spaces	200
Chapter 7. The Godbillon-Vey Classes	209
§7.1. The Godbillon Class and Measure Theory	209
§7.2. Proper Foliations	232
§7.3. Codimension One	234
§7.4. Quasi-polynomial Leaves	239
Part 3. Foliated 3-Manifolds	
Foreword to Part 3	251
Chapter 8. Constructing Foliations	253
§8.1. Orientable 3-Manifolds	254

§8.2. Open Book Decompositions	261
§8.3. Nonorientable 3-Manifolds	262
§8.4. Raymond's Theorem	265
§8.5. Thurston's Construction	274
 Chapter 9. Reebless Foliations	 285
§9.1. Statements of Results	286
§9.2. Poincaré-Bendixson Theory and Vanishing Cycles	290
§9.3. Novikov's Exploding Disk	300
§9.4. Completion of the Proofs of Novikov's Theorems	307
§9.5. The Roussarie-Thurston Theorems	312
 Chapter 10. Foliations and the Thurston Norm	 325
§10.1. Compact Leaves of Reebless Foliations	326
§10.2. Knots, Links, and Genus	333
§10.3. The Norm on Real Homology	340
§10.4. The Unit Ball in the Thurston Norm	345
§10.5. Foliations without Holonomy	355
 Chapter 11. Disk Decomposition and Foliations of Link Complements	 361
§11.1. A Basic Example	361
§11.2. Sutured Manifolds	364
§11.3. Operations on Sutured Manifolds	367
§11.4. The Main Theorem	376
§11.5. Applications	385
§11.6. Higher Depth	397
 Appendix A. C^* -Algebras	 399
§A.1. Bounded Operators	399
§A.2. Measures on Hausdorff Spaces	400
§A.3. Hilbert Spaces	403
§A.4. Topological Spaces and Algebras	406
§A.5. C^* -Algebras	408
§A.6. Representations of Algebras	410
§A.7. The Algebra of Compact Operators	415
§A.8. Representations of $C_0(X)$	418
§A.9. Tensor Products	420
§A.10. Von Neumann Algebras	422

Appendix B. Riemannian Geometry and Heat Diffusion	425
§B.1. Geometric Concepts and Formulas	425
§B.2. Estimates of Geometric Quantities	428
§B.3. Basic Function Theory	432
§B.4. Regularity Theorems	433
§B.5. The Heat Equation	436
§B.6. Construction of the Heat Kernel	438
§B.7. Estimates for the Heat Kernel	445
§B.8. The Green Function	447
§B.9. Dirichlet Problem and Harmonic Measure	449
§B.10. Diffusion and Resolvent	453
Appendix C. Brownian Motion	461
§C.1. Probabilistic Concepts	461
§C.2. Construction of Brownian Motion	465
§C.3. The Markov Process	469
§C.4. Continuity of Brownian Paths	474
§C.5. Stopping Times	478
§C.6. Some Consequences of the Markov Property	481
§C.7. The Discrete Dirichlet and Poisson Problems	483
§C.8. Dynkin's Formula	486
§C.9. Local Estimates of Exit Times	492
Appendix D. Planar Foliations	497
§D.1. The Space of Leaves	497
§D.2. Basic Isotopies	501
§D.3. The Hausdorff Case	506
§D.4. Decomposing the Foliation	510
§D.5. Construction of the Diffeomorphism	513
Bibliography	527
Index	537