

Contents

Preface	xi
Chapter 1. Elliptic curves	1
1.1. Elliptic curves	1
1.2. The Mordell-Weil theorem	3
1.3. The Birch and Swinnerton-Dyer conjecture	6
1.4. L -functions	7
1.5. Some known results	8
Further results and references	9
Exercises	10
Chapter 2. Modular forms	13
2.1. Modular forms	13
2.2. Hecke operators	14
2.3. Atkin-Lehner theory	16
2.4. L -series	17
2.5. Eichler-Shimura theory	18
2.6. Wiles' theorem	20
2.7. Modular symbols	21
Further results and references	25
Exercises	26
Chapter 3. Heegner points on $X_0(N)$	29
3.1. Complex multiplication	29
3.2. Heegner points	33
3.3. Numerical examples	34
3.4. Properties of Heegner points	35
3.5. Heegner systems	36
3.6. Relation with the Birch and Swinnerton-Dyer conjecture	37
3.7. The Gross-Zagier formula	39
3.8. Kolyvagin's theorem	40
3.9. Proof of the Gross-Zagier-Kolyvagin theorem	40
Further results	41
Exercises	42
Chapter 4. Heegner points on Shimura curves	45
4.1. Quaternion algebras	46
4.2. Modular forms on quaternion algebras	47
4.3. Shimura curves	49
4.4. The Eichler-Shimura construction, revisited	50

4.5. The Jacquet-Langlands correspondence	50
4.6. The Shimura-Taniyama-Weil conjecture, revisited	51
4.7. Complex multiplication for $\mathcal{H}/\Gamma_{N^+, N^-}$	51
4.8. Heegner systems	52
4.9. The Gross-Zagier formula	53
References	54
Exercises	54
Chapter 5. Rigid analytic modular forms	57
5.1. p -adic uniformisation	57
5.2. Rigid analytic modular forms	60
5.3. p -adic line integrals	63
Further results	65
Exercises	65
Chapter 6. Rigid analytic modular parametrisations	67
6.1. Rigid analytic modular forms on quaternion algebras	67
6.2. The Čerednik-Drinfeld theorem	68
6.3. The p -adic Shimura-Taniyama-Weil conjecture	68
6.4. Complex multiplication, revisited	69
6.5. An example	70
6.6. p -adic L -functions, d'après Schneider-Iovita-Spiess	73
6.7. A Gross-Zagier formula	74
Further results	75
Exercises	75
Chapter 7. Totally real fields	79
7.1. Elliptic curves over number fields	79
7.2. Hilbert modular forms	80
7.3. The Shimura-Taniyama-Weil conjecture	82
7.4. The Eichler-Shimura construction for totally real fields	83
7.5. The Heegner construction	84
7.6. A preview of Chapter 8	85
Further results	86
Chapter 8. ATR points	87
8.1. Period integrals	87
8.2. Generalities on group cohomology	88
8.3. The cohomology of Hilbert modular groups	89
8.4. ATR points	93
References	95
Exercises	95
Chapter 9. Integration on $\mathcal{H}_p \times \mathcal{H}$	97
9.1. Discrete arithmetic subgroups of $\mathbf{SL}_2(\mathbb{Q}_p) \times \mathbf{SL}_2(\mathbb{R})$	98
9.2. Forms on $\mathcal{H}_p \times \mathcal{H}$	99
9.3. Periods	101
9.4. Some p -adic cocycles	104
9.5. Stark-Heegner points	105
9.6. Computing Stark-Heegner points	106

Further results	109
Exercises	109
Chapter 10. Kolyvagin's theorem	113
10.1. Bounding Selmer groups	114
10.2. Kolyvagin cohomology classes	117
10.3. Proof of Kolyvagin's theorem	121
References	122
Exercises	122
Bibliography	125