CONTENTS

1 Historical introduction 1

2 Quantized Dirac fields 7
- 2.1 Dirac equation 7
- 2.2 Representations of γ matrices 9
- 2.3 Products of γ matrices 11
- 2.4 Relativistic covariance 13
- 2.5 Helicity 17
- 2.6 Gauge transformations 17
- 2.7 Chirality 18
- 2.8 Solution of the Dirac equation 22
- 2.9 Quantization 31
- 2.10 Symmetry transformation of states 36
- 2.11 C, P, and T transformations 48
- 2.12 Wave packets 60
- 2.13 Finite normalization volume 63
- 2.14 Fierz transformations 64

3 The Standard Model 67
- 3.1 Electroweak Lagrangian 71
- 3.2 Electroweak interactions 75
- 3.3 Three generations 80
- 3.4 The Higgs mechanism 83
- 3.5 Fermion masses and mixing 88
- 3.6 Gauge bosons 97
- 3.7 Effective low-energy CC and NC Lagrangians 102

4 Three-generation mixing 106
- 4.1 Diagonalization of the mass matrix 107
- 4.2 Physical parameters in the mixing matrix 108
- 4.3 Parameterization of the mixing matrix 109
- 4.4 Degenerate masses 116
- 4.5 Mixing matrix with one vanishing element 118
- 4.6 CP violation 120
- 4.7 Rephasing invariants 124
- 4.8 Unitarity triangles 129
- 4.9 Conditions for CP violation 133

5 Neutrino interactions 135
- 5.1 Neutrino–electron interactions 136

5.2 Hadron decays 147
5.3 Neutrino–nucleon scattering 160

6 Massive neutrinos 180
6.1 Dirac masses 180
6.2 Majorana neutrinos 188
6.3 Mixing of three Majorana neutrinos 208
6.4 One-generation Dirac–Majorana mass term 216
6.5 Three-generation Dirac–Majorana mixing 229
6.6 Special cases 235
6.7 Majorana mass matrix 237

7 Neutrino oscillations in vacuum 245
7.1 Standard Derivation of the Neutrino Oscillation Probability 247
7.2 Antineutrino case 254
7.3 CPT, CP, and T transformations 256
7.4 Two-neutrino mixing 259
7.5 Types of neutrino oscillation experiments 261
7.6 Averaged transition probability 267
7.7 Large Δm^2 dominance 273
7.8 Active small Δm^2 277

8 Theory of neutrino oscillations in vacuum 283
8.1 Plane-wave approximation 284
8.2 Wave-packet treatment 299
8.3 Size of neutrino wave packets 311
8.4 Questions 316

9 Neutrino oscillations in matter 322
9.1 Effective potentials in matter 323
9.2 Evolution of neutrino flavors 329
9.3 The MSW effect 331
9.4 Slab approximation 339
9.5 Parametric resonance 341
9.6 Geometrical representation 343

10 Solar neutrinos 352
10.1 Thermonuclear energy production 353
10.2 Standard solar models 359
10.3 Model-independent constraints on solar neutrino fluxes 364
10.4 Homestake experiment 366
10.5 Gallium experiments 368
10.6 Water Cherenkov detectors 372
10.7 Vacuum oscillations 381
10.8 Resonant flavor transitions in the Sun 382
10.9 Regeneration of solar ν_e's in the Earth 386

10.10 Global fit of solar neutrino data 389

11 Atmospheric neutrinos 390
11.1 Flux of atmospheric neutrinos 393
11.2 Atmospheric neutrino experiments 416

12 Terrestrial neutrino oscillation experiments 428
12.1 Sensitivity 429
12.2 Reactor experiments 432
12.3 Accelerator experiments 443

13 Phenomenology of three-neutrino mixing 452
13.1 Neutrino oscillations in vacuum 453
13.2 Matter effects 465
13.3 Analysis of oscillation data 474

14 Direct measurements of neutrino mass 484
14.1 Beta decay 485
14.2 Pion and tau decays 493
14.3 Neutrinoless double-beta decay 494

15 Supernova neutrinos 511
15.1 Supernova types 512
15.2 Supernova rates 515
15.3 Core-collapse supernova dynamics 517
15.4 SN1987A 528
15.5 Neutrino mass 534
15.6 Neutrino mixing 535
15.7 Other neutrino properties 536
15.8 Future 537

16 Cosmology 540
16.1 Basic general relativity 540
16.2 Robertson–Walker metric 543
16.3 Dynamics of expansion 553
16.4 Matter-dominated Universe 560
16.5 Radiation-dominated Universe 562
16.6 Curvature-dominated Universe 563
16.7 Vacuum-dominated Universe 563
16.8 Thermodynamics of the early Universe 564
16.9 Entropy 569
16.10 Decoupling 572
16.11 Cosmic microwave background radiation 577

17 Relic neutrinos 586
17.1 Neutrino decoupling 587

17.2 Electron-positron annihilation 588
17.3 Neutrino temperature 589
17.4 Energy density of light massive neutrinos 590
17.5 Energy density of heavy neutrinos 591
17.6 Big-Bang nucleosynthesis 596
17.7 Large-scale structure formation 600
17.8 Global fits of cosmological data 612
17.9 Number of neutrinos 618
17.10 Neutrino asymmetry 621

Appendices

A Conventions, useful formulas, and physical constants 626
A.1 Conventions 626
A.2 Pauli matrices 628
A.3 Dirac matrices 629
A.4 Mathematical formulas 634
A.5 Physical constants 635

B Special relativity 637
B.1 The Lorentz group 637
B.2 Representations of the Lorentz group 643
B.3 The Poincaré group and its representations 646

C Lagrangian theory 649
C.1 Variational principle and field equations 649
C.2 Canonical quantization 650
C.3 Noether's theorem 650
C.4 Space-time translations 652
C.5 Lorentz transformations 653
C.6 Complex fields 653
C.7 Global gauge symmetry 654

D Gauge theories 657
D.1 General formulation of gauge theories 657
D.2 Quantum chromodynamics 662

E Feynman rules of the standard electroweak model 664
E.1 External lines 664
E.2 Internal lines 665
E.3 Vertices 666
E.4 Cross-sections and decay rates 668

Bibliography 671

Index 705