Contents

Introduction 1

Chapter 1. Preparatory Information 5

1.1. The Spaces $C(\mathcal{E})$ and $L_p(\mathcal{E})$ 5
1.2. Normed Linear Spaces 9
1.3. Properties of the Space $L_p(\mathcal{E})$ 19
1.4. Averaging of Functions According to Sobolev 27
1.5. Generalized Functions 30

Chapter 2. Trigonometric Polynomials 81

2.1. Theorems on Zeros. Linear Independence 81
2.2. Important Examples of Trigonometric Polynomials 84
2.3. The Trigonometric Interpolation Polynomial of Lagrange . . 88
2.4. The Interpolation Formula of M. Riesz 90
2.5. The Bernšteĭn's Inequality 92
2.6. Trigonometric Polynomials of Several Variables 93
2.7. Trigonometric Polynomials Relative to Certain Variables . . 95

Chapter 3. Entire Functions of Exponential Type, Bounded on $\mathbb{R}_n$. 98

3.1. Preparatory Material 98
3.2. Interpolation Formula 111
3.3. Inequalities of Different Metrics for Entire Functions of Exponential Type 122
3.4. Inequalities of Different Dimensions for Entire Functions of Exponential Type 131
3.5. Subspaces of Functions of Given Exponential Type 134
3.6. Convolutions with Entire Functions of Exponential Type . . 135

Chapter 4. The Function Classes W, H, B 141

4.1. The Generalized Derivative 141
4.2. Finite Differences and Moduli of Continuity 146
4.3. The Classes W, H, B 152
4.4. Representation of an Intermediate Derivate in Terms of a Derivative of Higher Order and the Function. Corollaries 163

4.5. More on Sobolev Averages 174
4.6. Estimate of the Increment Relative to a Direction 176
4.7. Completeness of the Spaces W, H, B 177
4.8. Estimates of the Derivative by the Difference Quotient . . . 180

Chapter 5. Direct and Inverse Theorems of the Theory of Approximation. Equivalent Norms 183

5.1. Introduction 183
5.2. Approximation Theorem 185
5.3. Periodic Classes 193
5.4. Inverse Theorems of the Theory of Approximations 200
5.5. Direct and Inverse Theorems on Best Approximations. Equivalent H-Norms 207
5.6. Definition of B-Classes with the Aid of Best Approximations. Equivalent Norms 217

Chapter 6. Imbedding Theorems for Different Metrics and Dimensions 231

6.1. Introduction 231
6.2. Connections among the Classes B, H, W 234
6.3. Imbedding of Different Metrics 236
6.4. Trace of a Function 238
6.5. Imbeddings of Different Dimensions 240
6.6. The Simplest Inverse Theorem on Imbedding of Different Dimensions 244
6.7. General Imbedding Theorem for Different Dimensions 247
6.8. General Inverse Imbedding Theorem 248
6.9. Generalization of the Imbedding Theorem for Different Metrics 252
6.10. Supplementary Information 255

Chapter 7. Transitivity and Unimprovability of Imbedding Theorems. Compactness 261

7.1. Transitive Properties of Imbedding Theorems 261
7.2. Inequalities with a Parameter ε. Multiplicative Inequalities 263
7.3. Boundary Functions in H_p^r. Unimprovability of Imbedding Theorems 267
7.4. More on Boundary Functions in H_p^r 270
7.5. Unimprovability of Inequalities for Mixed Derivatives . . . 273
7.6. Another Proof of the Unimprovability of Imbedding Theorems 274
7.7. Theorems on Compactness 279

Chapter 8. Integral Representations and Isomorphism of Isotropy Classes 289

8.1. The Bessel-MacDonald Kernels 289
8.2. The Isomorphism Classes W_p^l 294
8.3. Properties of the Bessel-MacDonald Kernel 296
8.4. Estimate of the Best Approximation for $I_r f$ 298
8.5. Multipliers Equal to Unity on a Region 300
8.6. de la Vallée Poussin Sums of a Regular Function 301
8.7. An Inequality for the Operation I_{-r} $(r > 0)$ over Functions of Exponential Type 308
8.8. Decomposition of a Regular Function into Series Relative to de la Vallée Poussin Sums 312
8.9. Representation of Functions of the Classes $B_{p\theta}^r$ in Terms of de la Vallée Poussin Series. Null Classes $(1 \leqq p \leqq \infty)$ 313
8.10. Series Relative to Dirichlet Sums $(1 < p < \infty)$ 316

Chapter 9. The Liouville Classes L 323

9.1. Introduction 323
9.2. Definitions and Basic Properties of the Classes L_p^r and L_p^r 325
9.3. Interrelationships among Liouville and other Classes 332
9.4. Integral Representation of Anisotropic Classes 335
9.5. Imbedding Theorems 358
9.6. Imbedding Theorem with a Limiting Exponent 368
9.7. Nonequivalence of the Classes B_p^r and L_p^r 373

Remarks 377

Literature 403

Index of Names 415

Subject Index 417