Contents

Preface *v*

Chapter 1. Possibility of Approximation

1. Basic Notions *1*
2. Linear Operators *3*
3. Approximation Theorems *7*
4. The Theorem of Stone *11*
5. Notes *13*

Chapter 2. Polynomials of Best Approximation

1. Existence of Polynomials of Best Approximation *16*
2. Characterization of Polynomials of Best Approximation *18*
3. Applications of Convexity *19*
4. Chebyshev Systems *23*
5. Uniqueness of Polynomials of Best Approximation *26*
6. Chebyshev's Theorem *29*
7. Chebyshev Polynomials *30*
8. Approximation of Some Complex Functions *32*
9. Notes *33*

Chapter 3. Properties of Polynomials and Moduli of Continuity

1. Interpolation *36*
2. Inequalities of Bernstein *39*
3. The Inequality of Markov *40*
4. Growth of Polynomials in the Complex Plane *42*
5. Moduli of Continuity *43*
6. Moduli of Smoothness *47*
7. Classes of Functions *49*
8. Notes *52*

Chapter 4. The Degree of Approximation by Trigonometric Polynomials

1. Generalities *54*
2. The Theorem of Jackson *55*
3. The Degree of Approximation of Differentiable Functions *57*
4. Inverse Theorems *58*
5. Differentiable Functions *61*
6. Notes *62*

Chapter 5. The Degree of Approximation by Algebraic Polynomials

1. Preliminaries *65*
2. The Approximation Theorems *68*
3. Inequalities for the Derivatives of Polynomials *69*
4. Inverse Theorems *73*
5. Approximation of Analytic Functions *76*
6. Notes *78*

Chapter 6. Approximation by Rational Functions. Functions of Several Variables

1. Degree of Rational Approximation *81*
2. Inverse Theorems *83*
3. Periodic Functions of Several Variables *87*
4. Approximation by Algebraic Polynomials *89*
5. Notes *90*

Chapter 7. Approximation by Linear Polynomial Operators

1. Sums of de la Vallée-Poussin. Positive Operators *92*
2. The Principle of Uniform Boundedness *95*
3. Operators that Preserve Trigonometric Polynomials *96*
4. Trigonometric Saturation Classes *98*
5. The Saturation Class of the Bernstein Polynomials *102*
6. Notes *108*

Chapter 8. Approximation of Classes of Functions

1. Introduction *111*
2. Approximation in the Space L^1 *112*
3. The Degree of Approximation of the Classes W_p^* *115*

4. Distance Matrices 120
5. Approximation of the Classes Λ_ω 122
6. Arbitrary Moduli of Continuity; Approximation by Operators 124
7. Analytic Functions 126
8. Notes 130

Chapter 9. Widths

1. Definitions and Basic Properties 132
2. Sets of Continuous and Differentiable Functions 133
3. Widths of Balls 137
4. Applications of Theorem 2 139
5. Differential Operators 143
6. Widths of the Sets $\mathfrak{R}_i$ 146
7. Notes 148

Chapter 10. Entropy

1. Entropy and Capacity 150
2. Sets of Continuous and Differentiable Functions 153
3. Entropy of Classes of Analytic Functions 155
4. More General Sets of Analytic Functions 161
5. Relations between Entropy and Widths 163
6. Notes 166

Chapter 11. Representation of Functions of Several Variables by Functions of One Variable

1. The Theorem of Kolmogorov 168
2. The Fundamental Lemma 170
3. The Completion of the Proof 173
4. Functions Not Representable by Superpositions 174
5. Notes 177

Bibliography 179

Index 187