Contents

CHAPTER SEVEN. VARISOLVENT FUNCTIONS

7–1 Introduction 1
7–2 Varisolvent functions 3
7–3 Existence of best approximations 9
7–4 Basic theorems 9
7–5 Approximation on subsets 12
7–6 The limits of a Tchebycheff type theory—characterization . . 15
7–7 The problem with assumption $\mathscr{A}1$ 17
7–8 The problem with assumption $\mathscr{A}2$ 22
7–9 The limits of a Tchebycheff type theory—characterization and existence 31
7–10 Areas of difficulty and unsolved problems 38

CHAPTER EIGHT. THE EXPONENTIAL FUNCTION AND RELATED FUNCTIONS

8–1 Introduction 42
8–2 The exponential approximating function 42
8–3 Approximation from a curve of functions 46
8–4 The existence theorems for L_p-approximation 48
8–5 Tchebycheff approximation and varisolvence 56
8–6 Concrete interpretations 63
8–7 Finite point sets and pseudo-approximations 65
8–8 Computation 69

CHAPTER NINE. RATIONAL FUNCTIONS

9–1 Introduction 76
9–2 Existence theorems for the interval [0, 1] 77
9–3 Stability and existence for general rational functions 84
9–4 Characterization and uniqueness for general rational functions . 89
9–5 Polynomial rational functions 97
9–6 Computation 102
9–7 Computation—the Remes algorithm 105
9–8 Computation—other methods 113

CHAPTER TEN. SPLINE FUNCTIONS

10–1 Introduction 123
10–2 Spline functions and their basic properties 124
10–3 Existence theorems for nonlinear spline approximation . . . 129
10–4 L_2-approximation by splines 142
10–5 Tchebycheff approximation by splines 145
10–6 Splines and the approximation of linear functionals 154
10–7 Computational methods 159

CHAPTER ELEVEN. GEOMETRIC THEORY OF NONLINEAR APPROXIMATION

11–1 Introduction 168
11–2 Approximation and projection 169
11–3 Unisolvent and locally unisolvent functions as manifolds . . . 171
11–4 Minkowski geometry 173
11–5 Curvature in Minkowski geometry 190
11–6 Motzkin sets and unique projections 198
11–7 Projections on submanifolds 201
11–8 Non-manifold problems 212
11–9 Projections on objects 217

CHAPTER TWELVE. MULTIVARIATE APPROXIMATION

12–1 Introduction 224
12–2 Least squares and general orthonormal systems 225
12–3 Tensor products 227
12–4 Tchebycheff approximation—nonuniqueness 230
12–5 Tchebycheff approximation—characterization 232
12–6 Conditions for nonuniqueness 236
12–7 Strict approximation 239
12–8 The Polya algorithm 246
12–9 Computation 251
12–10 Orthogonalization 251
12–11 The one for one exchange algorithm 254
12–12 Critical point sets 258

CHAPTER THIRTEEN. BEST APPROXIMATIONS AND INTERPOLATING FUNCTIONS

13–1 Introduction 260
13–2 Norms and regularity 263
13–3 Varisolvent approximating functions on an interval 267
13–4 Linear interpolating functions on an interval 273
13–5 Varisolvent approximations on a finite point set 278
13–6 Norm classes for finite point sets 279
13–7 Linear approximation and L_p-norms 283

13–8 The Tchebycheff norm 290
13–9 Summary for L_p-norms and linear functions 291
13–10 Juxtapolynomials 293
13–11 The Lawson algorithm 298

BIBLIOGRAPHY 309

INDEX 329