

# CONTENTS

*Foreword* xi

*Acknowledgments* xvii

*Introduction* 1

CHAPTER

1

## THE FOUNDATIONS 8

- 1.1. The 1920s: Sets 10
- 1.2. The 1940s: Structures 14
- 1.3. The 1960s: Categories 17
- 1.4. The 1980s: Functions 21

CHAPTER

2

## PURE MATHEMATICS 25

- 2.1. Mathematical Analysis: Lebesgue Measure (1902) 29
- 2.2. Algebra: Steinitz Classification of Fields (1910) 33
- 2.3. Topology: Brouwer's Fixed-Point Theorem (1910) 37
- 2.4. Number Theory: Gelfand Transcendental Numbers (1929) 39

- 2.5. Logic: Gödel's Incompleteness Theorem (1931) 43
- 2.6. The Calculus of Variations: Douglas's Minimal Surfaces (1931) 47
- 2.7. Mathematical Analysis: Schwartz's Theory of Distributions (1945) 52
- 2.8. Differential Topology: Milnor's Exotic Structures (1956) 56
- 2.9. Model Theory: Robinson's Hyperreal Numbers (1961) 59
- 2.10. Set Theory: Cohen's Independence Theorem (1963) 63
- 2.11. Singularity Theory: Thom's Classification of Catastrophes (1964) 66
- 2.12. Algebra: Gorenstein's Classification of Finite Groups (1972) 71
- 2.13. Topology: Thurston's Classification of 3-Dimensional Surfaces (1982) 78
- 2.14. Number Theory: Wiles's Proof of Fermat's Last Theorem (1995) 82
- 2.15. Discrete Geometry: Hales's Solution of Kepler's Problem (1998) 87

- 3.1. Crystallography: Bieberbach's Symmetry Groups (1910) 98
- 3.2. Tensor Calculus: Einstein's General Theory of Relativity (1915) 104
- 3.3. Game Theory: Von Neumann's Minimax Theorem (1928) 108
- 3.4. Functional Analysis: Von Neumann's Axiomatization of Quantum Mechanics (1932) 112

- 3.5. Probability Theory: Kolmogorov's Axiomatization (1933) 116
- 3.6. Optimization Theory: Dantzig's Simplex Method (1947) 120
- 3.7. General Equilibrium Theory: The Arrow-Debreu Existence Theorem (1954) 122
- 3.8. The Theory of Formal Languages: Chomsky's Classification (1957) 125
- 3.9. Dynamical Systems Theory: The KAM Theorem (1962) 128
- 3.10. Knot Theory: Jones Invariants (1984) 132

## CHAPTER

## 4

MATHEMATICS AND THE  
COMPUTER 139

- 4.1. The Theory of Algorithms: Turing's Characterization (1936) 145
- 4.2. Artificial Intelligence: Shannon's Analysis of the Game of Chess (1950) 148
- 4.3. Chaos Theory: Lorenz's Strange Attractor (1963) 151
- 4.4. Computer-Assisted Proofs: The Four-Color Theorem of Appel and Haken (1976) 154
- 4.5. Fractals: The Mandelbrot Set (1980) 159

## CHAPTER

## 5

## OPEN PROBLEMS 165

- 5.1. Arithmetic: The Perfect Numbers Problem (300 B.C.) 166
- 5.2. Complex Analysis: The Riemann Hypothesis (1859) 168

x CONTENTS

5.3. Algebraic Topology: The Poincaré Conjecture (1904)	172
5.4. Complexity Theory: The P = NP Problem (1972)	176
<i>Conclusion</i>	181
<i>References and Further Reading</i>	187
<i>Index</i>	189