

CONTENTS

Preface	xi
CHAPTER I. ALGEBRAIC PRELIMINARIES	1
1. Inner Product Spaces	1
2. Dual Vector Spaces	6
3. The Orthogonal Group (Algebraic Study)	11
4. Self-Adjoint Transformations and Bilinear Forms	20
5. Tensor Product of Vector Spaces	25
6. Euclidean Spaces	32
CHAPTER II. DIFFERENTIABLE STRUCTURES	37
1. Differentiable Mappings	37
2. Submanifolds of Euclidean Space	40
3. Matrix Lie Groups	42
4. Tangent Spaces	47
5. Cotangent Space	53
6. Vector Fields and Forms	57

7. Tensor Spaces, 2-Forms, and the d Operator	61
8. Analytic Machinery	68
CHAPTER III. MATRIX LIE GROUPS AND FRAME BUNDLES	74
1. Parallel Displacement and Group Parallelism	74
2. Induced Riemann Metric	82
3. Vector-Valued Forms	86
4. The Orthonormal Frame Bundle Over E^n	92
5. Left-Invariant Forms on Matrix Lie Groups	95
6. Equations of Structure of Matrix Lie Groups	100
7. Equations of Structure for \mathfrak{F}_0	101
CHAPTER IV. DIFFERENTIAL INVARIANTS OF SURFACES AND CURVES	110
1. Existence of Orthonormal Vector Fields	110
2. The Second Fundamental Form	113
3. The Second Fundamental Form (Continued)	119
4. The Frenet Formula	126
5. Uniqueness Theorems for Curves and Surfaces	133
6. Gaussian Curvature Zero and Development of Curves	137
CHAPTER V. LOCAL THEORY OF SURFACES	145
1. Orthonormal Frames in Coordinate Neighborhoods	145
2. General Frames and Affine Connections	152
3. The Classical Approach to Surface Theory	161
4. Geodesics Revisited	162
5. Elementary Geometric Theorems	171
6. Geodesic Coordinates and Applications	178
7. Examples	193

CHAPTER VI. GLOBAL STUDY OF SURFACES	199
1. Global Behavior of Geodesics	199
2. Complete Riemann Surfaces	203
3. Complete Surfaces of Constant Curvature	214
4. Abstract Riemann Manifolds	221
5. Hyperbolic Geometry	223
CHAPTER VII. INTEGRATION OF FORMS AND THE GAUSS-BONNET THEOREM	237
1. Partition of Unity	237
2. Integration of Forms on Surfaces	244
3. Rotation Index of a Vector Field	254
4. An Introduction to Theorems of the Gauss-Bonnet Type	265
Index	269