Contents

Preface **vii**

1 Euclidean Geometry **1**
§1.1. Euclid's Parallel Postulate 1
§1.2. Independence of the Parallel Postulate 7
§1.3. Euclidean n-Space 13
§1.4. Geodesics 22
§1.5. Arc Length 28
§1.6. Historical Notes 33

2 Spherical Geometry **36**
§2.1. Spherical n-Space 36
§2.2. Elliptic n-Space 42
§2.3. Spherical Arc Length 44
§2.4. Spherical Volume 45
§2.5. Spherical Trigonometry 48
§2.6. Historical Notes 54

3 Hyperbolic Geometry **56**
§3.1. Lorentzian n-Space 56
§3.2. Hyperbolic n-Space 63
§3.3. Hyperbolic Arc Length 77
§3.4. Hyperbolic Volume 80
§3.5. Hyperbolic Trigonometry 83
§3.6. Historical Notes 103

4 Inversive Geometry **105**
§4.1. Reflections 105
§4.2. Stereographic Projection 112
§4.3. Möbius Transformations 115
§4.4. Poincaré Extension 121
§4.5. The Conformal Ball Model 127
§4.6. The Upper Half-Space Model 136
§4.7. Classification of Transformations 141
§4.8. Historical Notes 146

5 Isometries of Hyperbolic Space **148**
§5.1. Topological Groups 148
§5.2. Groups of Isometries 154
§5.3. Discrete Groups 161
§5.4. Discrete Euclidean Groups 169
§5.5. Elementary Groups 180
§5.6. Historical Notes 189

6 Geometry of Discrete Groups **192**
§6.1. The Projective Disk Model 192
§6.2. Convex Sets 198
§6.3. Convex Polyhedra 205
§6.4. Polytopes 225
§6.5. Fundamental Domains 233
§6.6. Convex Fundamental Polyhedra 246
§6.7. Tessellations 251
§6.8. Historical Notes 260

7 Classical Discrete Groups **263**
§7.1. Reflection Groups 263
§7.2. Simplex Reflection Groups 277
§7.3. Generalized Simplex Reflection Groups 297
§7.4. Crystallographic Groups 305
§7.5. Torsion-Free Linear Groups 318
§7.6. Historical Notes 328

8 Geometric Manifolds **330**
§8.1. Geometric Spaces 330
§8.2. Clifford-Klein Space-Forms 337
§8.3. (X, G)-Manifolds 343
§8.4. Developing 350
§8.5. Completeness 357
§8.6. Curvature 367
§8.7. Historical Notes 369

9 Geometric Surfaces **371**
§9.1. Compact Surfaces 371
§9.2. Gluing Surfaces 374
§9.3. The Gauss-Bonnet Theorem 386
§9.4. Moduli Spaces 387
§9.5. Closed Euclidean Surfaces 397
§9.6. Closed Geodesics 400
§9.7. Closed Hyperbolic Surfaces 407
§9.8. Hyperbolic Surfaces of Finite Area 415
§9.9. Historical Notes 428

10 Hyperbolic 3-Manifolds **431**
§10.1. Gluing 3-Manifolds 431
§10.2. Complete Gluing of 3-Manifolds 440
§10.3. Finite Volume Hyperbolic 3-Manifolds 444
§10.4. Hyperbolic Volume 461
§10.5. Hyperbolic Dehn Surgery 475
§10.6. Historical Notes 500

11 Hyperbolic n-Manifolds **503**
§11.1. Gluing n-Manifolds 503
§11.2. Poincaré's Theorem 511
§11.3. Simplices of Maximum Volume 518
§11.4. The Gromov Invariant 529
§11.5. Measure Homology 539
§11.6. Mostow Rigidity 552
§11.7. Historical Notes 571

12 Geometrically Finite n-Manifolds **573**
§12.1. Limit Sets of Discrete Groups 573
§12.2. Limit Points of Discrete Groups 588
§12.3. Geometrically Finite Discrete Groups 603
§12.4. Nilpotent Groups 619
§12.5. The Margulis Lemma 628
§12.6. Geometrically Finite Manifolds 635
§12.7. Historical Notes 648

13 Geometric Orbifolds **652**
§13.1. Orbit Spaces 652
§13.2. (X, G)-Orbifolds 662
§13.3. Developing Orbifolds 671
§13.4. Gluing Orbifolds 694
§13.5. Poincaré's Theorem 709
§13.6. Historical Notes 713

Bibliography **715**

Index **736**