

# Contents

*Preface*

*xi*

## 1. Fundamental Definitions and Equations

1.1. The Numerical Problem and Nomenclature	1
1.2. Taylor Series Expansion	3
1.3. Aspects of Numerical Interpolation	4
1.4. Differentiation Formulas	7
1.5. Specific Integration Formulas	7
1.6. Integration Formulas for ODE	10
1.7. Generalized Integration Formulas for ODE	15
1.8. Compilation of Various Multiple-Step Integration Formulas Including $y_i$ and $y_i'$	22
1.9. Compilation of Various Multiple-Step Formulas Including $y_i$ and $y_i''$	27
1.10. Multiple-Step Integration Formulas: Higher Derivative Terms	28
1.11. Further Definitions	30
References	37

## 2. Runge–Kutta and Allied Single-Step Methods

2.1. Development of Single-Step Runge–Kutta Formulas	39
2.2. Condensed Nomenclature for Runge–Kutta Methods	45
2.3. Explicit Runge–Kutta Equations of Different Order	46

2.4. Runge–Kutta Formulas Derived from Truncation Error Analysis	56
2.5. Quadrature and Implicit Formulas	59
2.6. Runge–Kutta Formulas for Vector Differential Equations	65
2.7. Two-Step Runge–Kutta Formulas	67
2.8. Local Truncation Error Estimates in a Single Step	68
2.9. Local Truncation Error Estimate in Two Steps	74
2.10. Local Truncation Error Estimates in More than Two Steps	76
2.11. Local Round-Off Error in Single-Step Methods	77
2.12. The Explicit Use of Single-Step Formulas	78
2.13. Modern Taylor Series Expansions	79
2.14. Published Numerical Results	81
2.15. Numerical Experiments	83
References	103
<b>3. Stability of Multistep and Runge–Kutta Methods</b>	
3.1. Linear Multistep Methods	107
3.2. Numerical Stability of Linear Multistep Methods	109
3.3. Dahlquist Stability Theorems	116
3.4. Stability of Multistep Methods in Integrating Coupled ODEs	118
3.5. Stability of Integration of Nonlinear ODEs	128
3.6. Stability of Runge–Kutta Methods	129
3.7. Stability of Single-Step Methods Employing Second Derivatives	137
3.8. Stability versus Accuracy	140
3.9. Published Numerical Results	142
3.10. Numerical Experiments	143
References	150
<b>4. Predictor–Corrector Methods</b>	
4.1. A Simple Predictor–Corrector Set	153
4.2. A Modified Predictor–Corrector Set	155
4.3. Convergence of Iterations in the Corrector	157
4.4. Accuracy and Stability for Some Simple Predictor–Corrector Methods	162
4.5. Milne Predictor–Corrector Forms	166
4.6. Hamming Predictor–Corrector Set	174
4.7. Adams Predictor–Corrector Set	181
4.8. PE(CE) <sup>s</sup> versus P(EC) <sup>s</sup> Combinations	193
4.9. Special Second-Order Differential Equations	198
4.10. Use of Higher-Order Derivative P–C Combinations	198
4.11. Hybrid Type Methods	199
4.12. Starting the P–C Computation	208
4.13. Adjustment of the Step Size during the P–C Solution	208
4.14. Tabulation of Equations and Stability Bounds	210
4.15. Published Numerical Results	212
4.16. Numerical Experiments	215
4.17. Numerical Comparisons of Single- and Multiple-Step Methods	230
References	239

**5. Extrapolation Methods**

5.1. Extrapolation to the Limit	242
5.2. Extrapolation Algorithms for ODE	247
5.3. Stability and Error Analysis of Extrapolation Methods	253
5.4. Published Numerical Results	257
5.5. Numerical Experiments	258
References	266

**6. Numerical Integration of Stiff Ordinary Differential Equations**

6.1. Definition of the Problem	267
6.2. Explicit Single-Step Methods	270
6.3. Implicit Single-Step Methods	275
6.4. Predictor–Corrector Methods	281
6.5. Other Stiff Methods	283
6.6. Published Numerical Results	285
6.7. Numerical Experiments	286
References	292

*Index*

295