Contents

1.	INTRO	DDUCTION	1
	1.1	Ordinary Differential Equations	2
	1.2	Historical Remarks	5
2.	FIRST	ORDER DIFFERENTIAL EQUATIONS	11
	2.1	Linear Equations	11
	2.2	Further Discussion of Linear Equations	19
	2.3	Nonlinear Equations	24
	2.4	Separable Equations	31
	2.5	Exact Equations	36
	2.6	Integrating Factors	42
	2.7	Homogeneous Equations	45
	2.8	Miscellaneous Problems	49
	2.9	Applications of First Order Equations	51
	2.10	Elementary Mechanics	62
	*2.11	The Existence and Uniqueness Theorem	70
		Appendix. Derivation of Equation of Motion of a Body with Variable Mass	80
3.	SECON	ND ORDER LINEAR EQUATIONS	82
	3.1	Introduction	82
	3.2	Fundamental Solutions of the Homogeneous Equation	87
	3.3	Linear Independence	96
	3.4	Reduction of Order	99
	3.5	Homogeneous Equations with Constant Coefficients	103

xii	Contents

4.

5.

6.

3.5.1	Complex Roots	107
3.6	The Nonhomogeneous Problem	112
3.6.1	The Method of Undetermined Coefficients	115
3.6.2	The Method of Variation of Parameters	121
3.7	Mechanical Vibrations	127
3.7.1	Free Vibrations	130
3.7.2	Forced Vibrations	135
3.8	Electrical Networks	139
	S SOLUTIONS OF SECOND ORDER LINEAR TIONS	145
4.1	Introduction. Review of Power Series	145
4.2	Series Solutions Near an Ordinary Point, Part I	152
4.2.1	Series Solutions Near an Ordinary Point, Part II	161
4.3	Regular Singular Points	169
4.4	Euler Equations	174
4.5	Series Solutions Near a Regular Singular Point, Part I	179
4.5.1	Series Solutions Near a Regular Singular Point, Part II	186
*4.6	Series Solutions Near a Regular Singular Point; $r_1 = r_2$ and $r_1 - r_2 = N$	192
*4.7	Bessel's Equation	195
HIGH	ER ORDER LINEAR EQUATIONS	207
5.1	Introduction	207
5.2	General Theory of nth Order Linear Equations	208
5.3	Homogeneous Equations with Constant Coefficients	213
5.4	The Method of Undetermined Coefficients	219
5.5	The Method of Variation of Parameters	222
THE LAPLACE TRANSFORM 2		
6.1	Introduction. Definition of the Laplace Transform	226
6.2	Solution of Initial Value Problems	233
6.3	Step Functions	242

		Cont	ents xiii
	6.3.1	A Differential Equation with a Discontinuous Forcing Function	n 249
	6.4	Impulse Functions	252
	6.5	The Convolution Integral	257
	6.6	General Discussion and Summary	263
7.	SYSTI	EMS OF FIRST ORDER LINEAR EQUATIONS	265
	7.1	Introduction	265
	7.2	Solution of Linear Systems by Elimination	272
	7.3	Review of Matrices	277
	7.4	Systems of Linear Algebraic Equations; Linear Independence, Eigenvalues, Eigenvectors	287
	7.5	Basic Theory of Systems of First Order Linear Equations	299
	7.6	Homogeneous Linear Systems with Constant Coefficients	304
	7.7	Complex Eigenvalues	312
	7.8	Repeated Eigenvalues	317
	7.9	Fundamental Matrices	324
	7.10	Nonhomogeneous Linear Systems	329
8.	NUME	ERICAL METHODS	336
	8.1	Introduction	336
	8.2	The Euler or Tangent Line Method	338
	8.3	The Error	344
	8.4	An Improved Euler Method	351
	8.5	The Three-Term Taylor Series Method	356
	8.6	The Runge-Kutta Method	358
	8.7	Some Difficulties with Numerical Methods	363
	8.8	A Multistep Method	367
	8.9	Systems of First Order Equations	374
9.	NONLINEAR DIFFERENTIAL EQUATIONS AND		
	STABILITY		378
	9.1	Introduction	378
	9.2	Solutions of Autonomous Systems	386

XiV	Contents

9.3	The Phase Plane; Linear Systems	397
9.4	Stability; Almost Linear Systems	409
9.5	Competing Species and Predator-Prey Problems	421
9.6	Liapounov's Second Method	434
* 9.7	Periodic Solutions and Limit Cycles	443
		452
10.1	Introduction	452
10.2	Heat Conduction and Separation of Variables	453
10.3	Fourier Series	461
10.4	The Fourier Theorem	470
10.5	Even and Odd Functions	476
10.6	Solution of Other Heat Conduction Problems	483
10.7	The Wave Equation; Vibrations of an Elastic String	491
10.8	Laplace's Equation \	503
	Appendix A. Derivation of the Heat Conduction Equation	511
	Appendix B. Derivation of the Wave Equation	515
		519
11.1	Introduction	519
11.2	Linear Homogeneous Boundary Value Problems; Eigenvalues and Eigenfunctions	523
11.3	Sturm-Liouville Boundary Value Problems	531
11.4	Nonhomogeneous Boundary Value Problems	544
*11.5	Singular Sturm-Liouville Problems	560
*11.6	Further Remarks on the Method of Separation of Variables; A Bessel Series Expansion	568
*11.7	Series of Orthogonal Functions; Mean Convergence	574
ANS	WERS TO PROBLEMS	A-1
INDEX		
	9.4 9.5 9.6 *9.7 PARTAND 10.1 10.2 10.3 10.4 10.5 10.6 10.7 10.8 BOURSTUR 11.1 11.2 11.3 11.4 *11.5 *11.6	9.4 Stability; Almost Linear Systems 9.5 Competing Species and Predator-Prey Problems 9.6 Liapounov's Second Method *9.7 Periodic Solutions and Limit Cycles PARTIAL DIFFERENTIAL EQUATIONS AND FOURIER SERIES 10.1 Introduction 10.2 Heat Conduction and Separation of Variables 10.3 Fourier Series 10.4 The Fourier Theorem 10.5 Even and Odd Functions 10.6 Solution of Other Heat Conduction Problems 10.7 The Wave Equation; Vibrations of an Elastic String 10.8 Laplace's Equation \ Appendix A. Derivation of the Heat Conduction Equation Appendix B. Derivation of the Wave Equation BOUNDARY VALUE PROBLEMS AND STURM-LIOUVILLE THEORY 11.1 Introduction 11.2 Linear Homogeneous Boundary Value Problems; Eigenvalues and Eigenfunctions 11.3 Sturm-Liouville Boundary Value Problems 11.4 Nonhomogeneous Boundary Value Problems *11.5 Singular Sturm-Liouville Problems *11.6 Further Remarks on the Method of Separation of Variables; A Bessel Series Expansion *11.7 Series of Orthogonal Functions; Mean Convergence ANSWERS TO PROBLEMS