

TABLE OF CONTENTS

INTRODUCTION	v
PART 1. ORDINARY DIFFERENTIAL EQUATIONS	1
1.0. Ordinary Differential Equations	4
1.1. Example: Global Similarity Transformation, Invariance and Reduction to Quadrature	6
1.2. Simple Examples of Groups of Transformations; Abstract Definition	13
1.3. One-Parameter Group in the Plane	22
1.4. Proof That a One-Parameter Group Essentially Contains Only One Infinitesimal Transformation and Is Determined by It	31
1.5. Transformations; Symbol of the Infinitesimal Transformation U	36
1.6. Invariant Functions and Curves	46
1.7. Important Classes of Transformations	49
1.8. Applications to Differential Equations; Invariant Families of Curves	54
1.9. First-Order Differential Equations Which Admit a Group; Integrating Factor; Commutator	59
1.10. Geometric Interpretation of the Integrating Factor	67
1.11. Determination of First-Order Equations Which Admit a Given Group	71
1.12. One-Parameter Group in Three Variables; More Variables	74
1.13. Extended Transformation in the Plane	84
1.14. A Second Criterion That a First-Order Differential Equation Admits a Group	89
1.15. Construction of All Differential Equations of First-Order Which Admit a Given Group	92
1.16. Criterion That a Second-Order Differential Equation Admits a Group	101
1.17. Construction of All Differential Equations of Second-Order Which Admit a Given Group ...	107
1.18. Examples of Application of the Method	116

PART 2.	PARTIAL DIFFERENTIAL EQUATIONS	143
2.0.	Partial Differential Equations	143
2.1.	Formulation of Invariance for the Special Case of One dependent and Two Independent Variables	147
2.2.	Formulation of Invariance in General	156
2.3.	Fundamental Solution of the Heat Equation; Dimensional Analysis	167
2.4.	Fundamental Solutions of Heat Equation Global Affinity	170
2.5.	The Relationship Between the Use of Dimensional Analysis and Stretching Groups to Reduce the Number of Variables of a Partial Differential Equation	181
2.6.	Use of Group Invariance to Obtain New Solutions from Given Solutions	199
2.7.	The General Similarity Solution of the Heat Equation	206
2.8.	Applications of the General Similarity Solution of the Heat Equation	221
2.9.	-Axially-Symmetric Wave Equation	248
2.10.	Similarity Solutions of the One-Dimensional Fokker-Planck Equation	258
2.11.	The Green's Function for an Instantaneous Line Particle Source Diffusing in a Gravitational Field and Under the Influence of a Linear Shear Wind - An Example of a P.D.E. in Three Variables Invariant Under a Two-Parameter Group	275
2.12.	Infinite Parameter Groups - Derivation of the Poisson Kernel	282
2.13.	Far Field of Transonic Flow	286
2.14.	Nonlinear and Other Examples	295
2.15.	Construction of Partial Differential Equations Invariant Under a Given Multi-parameter Group	303
APPENDIX.	SOLUTION OF QUASILINEAR FIRST-ORDER PARTIAL DIFFERENTIAL EQUATIONS	318
BIBLIOGRAPHY. PART 1		324
BIBLIOGRAPHY. PART 2		325
INDEX		327