Contents

Introduction 1

I. Differential equations in two variables with entire coefficients 9
§ 1. A representation of solutions of partial differential equations 10
§ 2. The integral operator of the first kind 12
§ 3. Further representations of integral operators 15
§ 4. A representation of the operator of the first kind in terms of integrals 17
§ 5. Properties of the integral operator of the first kind 19
§ 6. Some further properties of the integral operator of the first kind . . 20
§ 7. The differential equation $\Delta_2 V + F(r^2)\, V = 0$ 27
§ 8. Integral operators of exponential type 31
§ 9. The differential equation $\Delta_2 \psi + N(x)\, \psi = 0$ 33
§ 10. Differential equations of higher order 35

II. Harmonic functions in three variables 38
§ 1. Preliminaries 38
§ 2. Characteristic space $\mathfrak{C}_3$ 39
§ 3. Harmonic functions with rational $\mathbf{B}_3$-associates 43
§ 4. Period functions 50
§ 5. Relations between coefficients of a series development of a harmonic function and its singularities 54
§ 6. Another type of integral representations of harmonic functions . . . 57
§ 7. The behavior in the large of functions of the class $S(E, \zeta_0, \zeta_1)$ with a rational associate $f(\zeta)$ 60

III. Differential equations in three variables 63
§ 1. An integral operator generating solutions of the equation $\Delta_3 \psi + A(r^2)\, \mathbf{N} \cdot \nabla \psi + C(r^2)\, \psi = 0$ 64
§ 2. A series expansion for solutions of the equation $\Delta_3 \psi + A(r^2)\, \mathbf{N} \cdot \nabla \psi + C(r^2)\, \psi = 0$ 66
§ 3. An integral operator generating solutions of the equation $\Delta_3 \psi + F(y, z)\, \psi = 0$ 68
§ 4. A second integral operator generating solutions of the equation $\Delta_3 \psi + F(y, z)\, \psi = 0$ 71
§ 5. An integral operator generating solutions of the equation $\psi_x + \psi_{yy} + \psi_{zz} + F(y, z)\, \psi = 0$ 74
§ 6. An integral operator generating solutions of the equation $g^{\mu\nu} \nabla_\mu \nabla_\nu \varphi + h^\mu \nabla_\mu \varphi + k\varphi = 0$ 78

IV. Systems of differential equations 81
§ 1. Harmonic vectors of three variables. Preliminaries 81
§ 2. Harmonic vectors in the large and their representation as integrals . 83
§ 3. Integrals of harmonic vectors 86
§ 4. Relations between integrals of algebraic harmonic vectors in three variables and integrals of algebraic functions of a complex variable 89
§ 5. Generalization of the residue theorems to the case of the equation $\Delta_3\psi + F(r^2)\,\psi = 0$ 92
§ 6. An operator generating solutions of a system of partial differential equations 96

V. Equations of mixed type and elliptic equations with singular and non-analytic coefficients 106
§ 1. Introduction. The simplified case of equations of mixed type 106
§ 2. A generalization of the representation (1.12) of solutions of the equation (1.6) 108
§ 3. The operator (1.11b) in the general case 112
§ 4. Generating functions analogous to solutions of the hypergeometric equation 117
§ 5. On the solution of the initial value problem in the large. 120
§ 6. Generalized CAUCHY-RIEMANN equations 122
§ 7. The differential equation $\Delta_3\psi + N(x)\psi = 0$ with a new type of singularity of N 125
§ 8. An integral operator for equations with non-analytic coefficients . . 127

Bibliography 132
Subject Index 145