Contents

| 1. | INTRO | DUCTION | 1 |
|----|-------|--|----|
| | 1.1 | Ordinary Differential Equations | 2 |
| 2. | FIRST | ORDER DIFFERENTIAL EQUATIONS | 7 |
| | 2.1 | Linear Equations | 7 |
| | 2.2 | Further Discussion of Linear Equations | 14 |
| | 2.3 | Nonlinear Equations | 18 |
| | 2.4 | Separable Equations | 26 |
| | 2.5 | Exact Equations | 31 |
| | 2.6 | Integrating Factors | 35 |
| | 2.7 | Homogeneous Equations | 39 |
| | 2.8 | Miscellaneous Problems | 43 |
| | 2.9 | Applications of First Order Equations | 44 |
| | 2.10 | Elementary Mechanics | 55 |
| | *2.11 | The Existence and Uniqueness Theorem | 61 |
| | *2.12 | The Existence Theorem from a More Modern Viewpoint | 71 |
| 3. | SECO | ND ORDER LINEAR EQUATIONS | 75 |
| | 3.1 | Introduction | 75 |
| | 3.2 | Fundamental Solutions of the Homogeneous Equation | 80 |
| | 3.3 | Linear Independence | 89 |
| | 3.4 | Reduction of Order | 92 |
| | 3.5 | Homogeneous Equations with Constant Coefficients | 95 |
| | | | |

| | 3.5.1 | Complex Roots | 99 |
|----|---|---|---|
| | 3.6 | The Nonhomogeneous Problem | 103 |
| | 3.6.1 | The Method of Undetermined Coefficients | 106 |
| | 3.6.2 | The Method of Variation of Parameters | 113 |
| | 3.7 | Mechanical Vibrations | 118 |
| | 3.7.1 | Free Vibrations | 122 |
| | 3.7.2 | Forced Vibrations | 127 |
| 4. | SERIES SOLUTIONS OF SECOND ORDER LINEAR EQUATIONS | | 134 |
| | 4.1 | Introduction. Review of Power Series | 134 |
| | 4.2 | Series Solutions near an Ordinary Point, Part I | 138 |
| | 4.2.1 | Series Solutions near an Ordinary Point, Part II | 145 |
| | 4.3 | Regular Singular Points | 153 |
| | 4.4 | Euler Equations | 157 |
| | 4.5 | Series Solutions near a Regular Singular Point, Part I | 162 |
| | *4.5.1 | Series Solutions near a Regular Singular Point, Part II | 168 |
| | *4.6 | Bessel's Equation | 174 |
| 5. | THOTH | ER ORDER LINEAR EQUATIONS | |
| ٥. | HIGHI | EN ORDER ENGER EQUATIONS | 185 |
| ٥. | 5.1 | Introduction | 185 185 |
| ٥. | | | |
| ٥. | 5.1 | Introduction | 185 |
| ٥. | 5.1 5.2 | Introduction General Theory of <i>n</i> th Order Linear Equations | 185 187 |
| ٥. | 5.1 5.2 5.3 | Introduction General Theory of <i>n</i> th Order Linear Equations The Homogeneous Equation with Constant Coefficients | 185 187 191 |
| 6. | 5.1 5.2 5.3 5.4 5.5 | Introduction General Theory of <i>n</i> th Order Linear Equations The Homogeneous Equation with Constant Coefficients The Method of Undetermined Coefficients | 185 187 191 198 |
| | 5.1 5.2 5.3 5.4 5.5 | Introduction General Theory of <i>n</i> th Order Linear Equations The Homogeneous Equation with Constant Coefficients The Method of Undetermined Coefficients The Method of Variation of Parameters | 185 187 191 198 201 |
| | 5.1 5.2 5.3 5.4 5.5 SYSTE | Introduction General Theory of nth Order Linear Equations The Homogeneous Equation with Constant Coefficients The Method of Undetermined Coefficients The Method of Variation of Parameters MS OF FIRST ORDER LINEAR EQUATIONS | 185 187 191 198 201 205 |
| | 5.1 5.2 5.3 5.4 5.5 SYSTE 6.1 | Introduction General Theory of nth Order Linear Equations The Homogeneous Equation with Constant Coefficients The Method of Undetermined Coefficients The Method of Variation of Parameters MS OF FIRST ORDER LINEAR EQUATIONS Introduction | 185 187 191 198 201 205 205 |
| | 5.1 5.2 5.3 5.4 5.5 SYSTE 6.1 6.2 | Introduction General Theory of nth Order Linear Equations The Homogeneous Equation with Constant Coefficients The Method of Undetermined Coefficients The Method of Variation of Parameters MS OF FIRST ORDER LINEAR EQUATIONS Introduction Solution of Linear Systems by Elimination | 185 187 191 198 201 205 205 209 |
| | 5.1 5.2 5.3 5.4 5.5 SYSTE 6.1 6.2 6.3 | Introduction General Theory of nth Order Linear Equations The Homogeneous Equation with Constant Coefficients The Method of Undetermined Coefficients The Method of Variation of Parameters MS OF FIRST ORDER LINEAR EQUATIONS Introduction Solution of Linear Systems by Elimination Review of Matrices | 185 187 191 198 201 205 205 209 214 |
| | 5.1 5.2 5.3 5.4 5.5 SYSTE 6.1 6.2 6.3 6.4 | Introduction General Theory of nth Order Linear Equations The Homogeneous Equation with Constant Coefficients The Method of Undetermined Coefficients The Method of Variation of Parameters MS OF FIRST ORDER LINEAR EQUATIONS Introduction Solution of Linear Systems by Elimination Review of Matrices Basic Theory of Systems of First Order Linear Equations | 185 187 191 198 201 205 205 209 214 226 |

| | | Contents | xiii |
|--------|--|----------|------|
| 7. NUM | ERICAL METHODS | | 247 |
| 7.1 | Introduction | | 247 |
| 7.2 | The Euler or Tangent Line Method | | 249 |
| 7.3 | The Error | | 257 |
| 7.4 | An Improved Euler Method | | 264 |
| 7.5 | The Three-Term Taylor Series Method | | 269 |
| 7.6 | The Runge-Kutta Method | | 272 |
| 7.7 | Some Difficulties with Numerical Methods | | 276 |
| ANSWER | S TO PROBLEMS | | A-1 |
| INDEX | | | I-1 |