

# Contents

Preface	
1. Notation	1
2. Statement of Main Results	5
Definition 2.1: SE and SE(j)	6
Theorem 2.2: Extremal Errors and Extremal Polynomials	8
Theorem 2.3: Christoffel Functions on $\mathbb{R}$	10
Theorem 2.4: Orthonormal Polynomials on $\mathbb{R}$	12
Theorem 2.5: Largest Zeros of Extremal Polynomials	14
Theorem 2.6: Markov–Bernstein Inequalities	15
3. Preliminaries	16
4. Potentials and Majorisation Functions	27
5. Infinite–Finite Range Inequalities	45
Theorem 5.1: The $L_\infty$ Case	45
Theorem 5.2: The $L_p$ Case	46
6. Entire Functions	52
Theorem 6.1: Asymptotic Behaviour of $G_Q$	52
7. Largest Zeros of Extremal Polynomials	70
Theorem 7.1: Largest Zeros and Proof of Theorem 2.5	70
Corollary 7.2: Points of Equioscillation	70
Corollary 7.3: Markov–Bernstein Inequalities; Theorem 2.6	71
8. Approximation with Weighted Polynomials	79
Theorem 8.1: Approximation by $P_n(x)/H(a_n x)$	79

9. One-Sided Weighted Polynomial Approximation	89
Theorem 9.1: One-sided Approximation by $P_n(x)/H(c_n x)$	89
10. One-Sided Weighted Polynomial Approximation, II	103
Theorem 10.1: One-sided Approximation by $P_n(x)W(c_n x)$	103
Lemma 10.2: Relative Approximation of $1/(G_{Q/2}W)$	104
11. Explicit Extremal Polynomials, Christoffel Functions and Szegő Functions	111
12. Extremal Errors and Extremal Polynomials	130
Theorem 12.1: Extremal Errors; Theorem 2.2(a)	130
Theorem 12.2: Extremal Polynomials; Theorem 2.2(b)	132
13. Asymptotics of Orthogonal Polynomials on $\mathbb{R}$	136
Theorem 13.1: $1/G_{Q/2}^2$ and $[-\pi, \pi]$	136
Theorem 13.2: $W^2$ and $[-\pi, \pi]$	138
Theorem 13.4: $W^2$ and $\mathbb{R}$ ; Theorem 2.4	143
Theorem 13.7: $1/G_{Q/2}^2$ and $\mathbb{R}$	149
14. Mean Asymptotics of Extremal Polynomials	150
Theorem 14.3: Mean Asymptotics; Theorem 2.2(c)	153
Theorem 14.5: Extremal Polynomials; Theorem 2.2(b)	158
15. Christoffel Functions	160
Theorem 15.1: $H^{-2}$ and $c_n$	160
Theorem 15.3: $H^{-2}$ and $a_n$	166
Theorem 15.5: $W^2$	175
References	179
Appendix: Freud Weights	182
I. Main Results	183
Definition I.1: $FR_\Lambda$ and $FR_\Lambda(j)$	183

Theorem I.2: Christoffel Functions	183
Theorem I.3: Christoffel Functions	185
Theorem I.4: Orthogonal Polynomials	187
II. Preliminaries	189
III. Approximation with Weighted Polynomials	194
Theorem III.1: Approximation by $P_n(x)/H(a_n x)$	194
IV. One-Sided Weighted Polynomial Approximation	200
Theorem IV.1: One-sided Approximation by $P_n(x)/H(c_n x)$	200
V. Asymptotics of Orthogonal Polynomials on $\mathbb{R}$	209
Theorem V.1: $H^{-2}$ and $[-\pi, \pi]$	209
Lemma V.2: Relative Approximation of $1/(G_{Q/2}W)$	212
Theorem V.3: $W$ and $[-\pi, \pi]$	214
Theorem V.5: $W$ and $\mathbb{R}$	215
Theorem V.7: $H^{-1}$ and $\mathbb{R}$	219
VI. Christoffel Functions	220
Theorem VI.1: $H^{-2}$ and $c_n$	220
Theorem VI.3: $H^{-2}$ and $a_n$	226
Theorem VI.5: $W^2$	234
Index	238