

Contents

	Page
Preface	v
Part I: PRESENTATION OF NUMERICAL METHODS	1
1. <u>Finite-Difference Methods for Boundary-Value Problems</u>	3
1.1. Sample Problems	3
1.2. Finite-Difference Methods for Linear, Second Order Ordinary Differential Equations	5
1.3. A Finite-Difference Approximation for the Cantilevered Beam Problem	10
1.4. Finite-Difference Methods for a Nonlinear Boundary-Value Problem	13
1.5. Finite-Difference Approximations for Two-Dimensional Elliptic Equations	16
References	19
2. <u>Projection Methods for Variational Equations</u>	20
2.1. Basic Properties of Variational Equations and Sample Problems	21
2.2. Sample Problems (Revised)	28
2.3. The Ritz Method	35
2.4. Galerkin Methods and the Method of Least Squares	38
2.5. Projection Methods for Nonlinear Problems	44
References	50
3. <u>Approximation Methods for Integral Equations of the Second Kind</u>	51
3.1. Linear Integral Equations of the Second Kind	51
3.2. Quadrature Methods	56
3.3. Projection Methods	60
3.4. Approximations of Nonlinear Integral Equations	66
References	73
4. <u>Approximation Methods for Initial Value Problems in Partial Differential Equations</u>	74
4.1. Difference Methods for the Heat Equation	75
4.2. Galerkin Methods for the Heat Equation	85
4.3. Numerical Methods for the Wave Equation	91
4.4. The Numerical Solution of Nonlinear Initial Value Problems	101
4.5. Pure Initial Value Problems and a General Representation of Approximation Methods	109
References	119

	Page
Part II: CONVERGENCE THEORY	121
5. <u>The Concepts of Discrete Convergence and Discrete Approximations</u>	123
5.1. Definitions, Basic Properties, and First Examples	124
5.2. Restriction and Embedding Operators	127
5.3. Discrete Uniform Convergence of Continuous Functions	132
5.4. Discrete Approximations of L^p -Spaces and Weak Convergence of Measures	140
References	151
6. <u>Discrete Convergence of Mappings and Solutions of Equations</u>	152
6.1. Continuity and Differentiability of Mappings and Their Inverses	153
6.2. Stability and Inverse Stability of Sequences of Mappings	161
6.3. Consistency and Discrete Convergence of Mappings	168
6.4. Discrete Convergence of Solutions and Biconvergence	173
References	179
7. <u>Compactness Criteria for Discrete Convergence</u>	181
7.1. Discrete Compact Sequences of Elements	182
7.2. A-Regular and Regularly Convergent Mappings	186
7.3. Discrete Compact Sequences of Mappings and Biconvergence for Equations of the Second Kind	194
7.4. Projection Methods for the Approximate Solution of Nonlinear Fixed Point Equations	202
References	206
Part III: CONVERGENCE ANALYSIS FOR APPROXIMATE SOLUTIONS OF BOUNDARY-VALUE PROBLEMS AND INTEGRAL EQUATIONS	207
8. <u>Convergence of Finite-Difference Methods for Boundary-Value Problems</u>	209
8.1. Convergence of Difference Methods for Ordinary Differential Equations Via Maximum Principles	210
8.2. Convergence of Difference Methods for Ordinary Differential Equations Via Compactness Arguments	224
8.3. Convergence of the Five-Point Difference Approximation for Poisson's Equation	232
References	235
9. <u>Biconvergence for Projection Methods Via Variational Principles</u>	236
9.1. Approximability	237
9.2. Stability, Inverse Stability, and Biconvergence for Linear Problems	238
9.3. Biconvergence for Nonlinear Problems	244
References	250

	Page
10. <u>Convergence of Perturbations of Integral Equations of The Second Kind</u>	251
10.1. Statement of the Problem and Consistency	252
10.2. Equidifferentiability	257
10.3. Biconvergence	262
References	265
Part IV: INVERSE STABILITY, CONSISTENCY AND CONVERGENCE FOR INITIAL VALUE PROBLEMS IN PARTIAL DIFFERENTIAL EQUATIONS	266
11. <u>Inverse Stability and Convergence for General Discrete-Time Approximations of Linear and Nonlinear Initial Value Problems</u>	268
11.1. Statement of the Problem and Differentiability	270
11.2. Inverse Stability	287
11.3. Consistency and Convergence	294
References	305
12. <u>Special Criteria for Inverse Stability</u>	306
12.1. Linear Finite-Difference Methods with Positivity Properties	307
12.2. The von Neumann Condition	319
12.3. Inverse Stability of Galerkin Methods	335
12.4. Inverse Stability of Nonlinear Methods	342
References	353
13. <u>Convergence Analysis of Special Methods</u>	354
13.1. Consistency and Convergence of Finite-Difference Approximations	355
13.2. Consistency and Error Analysis of Discrete-Time Galerkin Methods	368
References	384
BIBLIOGRAPHY	385
GLOSSARY OF SYMBOLS	393
SUBJECT INDEX	394