

Contents

Preface vi *Introduction* ix *Notation* xiii

1	Measure and dimension	1
1.1	Basic measure theory	1
1.2	Hausdorff measure	7
1.3	Covering results	10
1.4	Lebesgue measure	12
1.5	Calculation of Hausdorff dimensions and measures	14
2	Basic density properties	20
2.1	Introduction	20
2.2	Elementary density bounds	22
3	Structure of sets of integral dimension	28
3.1	Introduction	28
3.2	Curves and continua	28
3.3	Density and the characterization of regular 1-sets	40
3.4	Tangency properties	46
3.5	Sets in higher dimensions	50
4	Structure of sets of non-integral dimension	54
4.1	Introduction	54
4.2	s -sets with $0 < s < 1$	54
4.3	s -sets with $s > 1$	57
4.4	Sets in higher dimensions	63
5	Comparable net measures	64
5.1	Construction of net measures	64
5.2	Subsets of finite measure	67
5.3	Cartesian products of sets	70
6	Projection properties	75
6.1	Introduction	75
6.2	Hausdorff measure and capacity	75

6.3	Projection properties of sets of arbitrary dimension	80
6.4	Projection properties of sets of integral dimension	84
6.5	Further variants	92
7	Besicovitch and Kakeya sets	95
7.1	Introduction	95
7.2	Construction of Besicovitch and Kakeya sets	96
7.3	The dual approach	101
7.4	Generalizations	106
7.5	Relationship with harmonic analysis	109
8	Miscellaneous examples of fractal sets	113
8.1	Introduction	113
8.2	Curves of fractional dimension	113
8.3	Self-similar sets	118
8.4	Osculatory and Apollonian packings	125
8.5	An example from number theory	131
8.6	Some applications to convexity	135
8.7	Attractors in dynamical systems	138
8.8	Brownian motion	145
8.9	Conclusion	149
	<i>References</i>	150
	<i>Index</i>	161