

# Contents

<i>Preface</i>	ix
<i>Acknowledgments</i>	xiii
<i>Notation</i>	xv

## Chapter 1 Sequences and Series

1.1 Order Symbols and Asymptotic Scales, Continuous Variables	1
1.2 Integer Variables	2
1.3 Sequences and Transformations in Abstract Spaces	3
1.4 Properties of Complex Sequences	5
1.5 Further Properties of Complex Sequences	8
1.6 Totally Monotone and Totally Oscillatory Sequences	12
1.7 Birkhoff–Poincaré Logarithmic Scales	15

## Chapter 2 Linear Transformations

2.1 Toeplitz's Theorem in a Banach Space	24
2.2 Complex Toeplitz Methods	27
2.3 Important Triangles	33
2.4 Toeplitz Methods Applied to Series of Variable Terms; Fourier Series and Lebesgue Constants	48
2.5 Toeplitz Methods and Rational Approximations; The Padé Table	53
2.6 Other Orthogonal Methods; Pollaczek Polynomials and Padé Approximants	59
2.7 Other Methods for Generating Toeplitz Transformations	63

**Chapter 3 Linear Lozenge Methods**

3.1	Background: Richardson Extrapolation and Romberg Integration	67
3.2	General Deltoids	71
3.3	Deltoids Obtained by Extrapolation	73
3.4	Example: Quadrature Based on Cardinal Interpolation	77
3.5	General Rhombus Lozenges	80

**Chapter 4 Optimal Methods and Methods Based on Power Series**

4.1	Best Methods for Laplace Moment Sequences	84
4.2	Optimal Approximations in $\ell^1$ and $\mathcal{R}_c$	90
4.3	Methods Based on Power Series	94

**Chapter 5 Nonlinear Lozenges; Iteration Sequences**

5.1	General Theory of Nonlinear Lozenge Algorithms	101
5.2	Path Regularity for Certain Lozenges	105
5.3	Iteration Sequences	106

**Chapter 6 The Schmidt Transformation; The  $\varepsilon$ -Algorithm**

6.1	Background	120
6.2	Derivation	121
6.3	Exactness Results	123
6.4	The Effect of $e_k$ on Certain Series	126
6.5	Power Series and $e_k$ ; The Padé Table	128
6.6	Geometrical Significance of the Schmidt Transformation	136
6.7	The $\varepsilon$ -Algorithm	138
6.8	The Stability of the $\varepsilon$ -Algorithm	141
6.9	Rational Analogs of the Formulas of Numerical Analysis	142
6.10	Generalizations of the $\varepsilon$ -Algorithm	144
6.11	Fixed Points of Differentiable Functions	146

**Chapter 7 Aitken's  $\delta^2$ -Process and Related Methods**

7.1	Aitken's $\delta^2$ -Process	149
7.2	The Lubkin $W$ -Transform	152
7.3	Related Algorithms	154

**Chapter 8 Lozenge Algorithms and the Theory of Continued Fractions**

8.1	Background	156
8.2	The Quotient Difference Algorithm; The $\eta$ -Algorithm	156

**Chapter 9 Other Lozenge Algorithms and Nonlinear Methods**

9.1	A Multiparameter $\varepsilon$ -Algorithm	166
9.2	The $\rho$ -Algorithm	168
9.3	The $\theta$ -Algorithm	169
9.4	Implicit Summation: Logarithmically Convergent Sequences	171

**Chapter 10 The Brezinski–Håvie Protocol**

10.1	Introduction and Derivation; Sequences in a Banach Space	175
10.2	The Case $\phi$ Constant	180
10.3	The Topological Schmidt Transformation	182
10.4	The Scalar Case	185
10.5	The Levin Transformations	189
10.6	Special Computational Procedures: The Trench Algorithm	198

**Chapter 11 The Brezinski–Håvie Protocol and Numerical Quadrature**

11.1	Introduction; The $G$ -Transform	200
11.2	The Computation of Fourier Coefficients	205
11.3	The tanh Rule	207

**Chapter 12 Probabilistic Methods**

12.1	Introduction	210
12.2	Derivation of the Methods	211
12.3	Properties of the Methods	216
12.4	Numerics	223

**Chapter 13 Multiple Sequences**

13.1	Rectangular Transformations	227
13.2	Crystal Lattice Sums	232

**Appendix**

A.1	Lagrangian Interpolation	243
A.2	The Formula for the $\varepsilon$ -Algorithm	244
A.3	Sylvester's Expansion Theorem	247

<b>Bibliography</b>	249
---------------------	-----

<b>Index</b>	255
--------------	-----