Contents

| Preface | ix |
|--|----------|
| Summary of Notation | xi |
| CHAPTER I. Introductory Concepts and Some General Results | 1 |
| §1. Types of convergence | 1 |
| §2. Completeness, totality, biorthogonality | 5 |
| §3. Fourier coefficients and partial sums of an orthogonal series | |
| §4. The basis property | 9 |
| CHAPTER II. Independent Functions and Their First Applications §1. Definition and construction of sequences of independent | 17 |
| functions | 17 |
| §2. Properties of systems of independent functions | 24 |
| §3. Convergence for almost all choices of signs, and | |
| unconditional convergence | 38 |
| §4. Random permutations | 51 |
| CHAPTER III. The Haar System | 61 |
| §1. Definition; form of the partial sums | 61 |
| §2. Inequalities for coefficients and theorems on the convergence | . |
| of Fourier-Haar series | 64 |
| §3. Unconditional convergence of Fourier-Haar series | |
| in $L^p(0,1)$ | 71 |
| §4. Convergence almost everywhere and in measure of Fourier | |
| series in the Haar system | 87 |
| §5. Absolute convergence almost everywhere and unconditional | |
| convergence almost everywhere for Haar series | 93 |
| §6. Transformations of the Haar system | 100 |

v

vi CONTENTS

| CHAPTER IV. Some Results on the Trigonometric and Walsh | |
|---|-----|
| Systems | 105 |
| §1. Properties of the partial sums of Fourier series, Fourier | |
| coefficients, and Fejér means | 105 |
| §2. Best approximation. Vallée-Poussin means | 110 |
| §3. Convergence of trigonometric series in L^p and almost | |
| everywhere | 114 |
| §4. Uniform and absolute convergence of Fourier series | 122 |
| §5. The Walsh system. Definition and some properties | 134 |
| CHAPTER V. The Hilbert Transform and some Function Spaces | 145 |
| §1. The Hilbert transform | 145 |
| §2. The spaces $Re \mathcal{H}^1$ and BMO | 160 |
| §3. The spaces $\mathcal{H}(\Delta)$ and BMO(Δ) (nonperiodic case) | 172 |
| CHAPTER VI. The Faber-Schauder and Franklin Systems | 185 |
| §1. The Faber-Schauder system | 185 |
| §2. Systems of Faber-Schauder type | 195 |
| §3. The Franklin system. Definition, elementary properties | 197 |
| §4. The exponential inequality for the Franklin functions | 201 |
| §5. Unconditional convergence of Fourier-Franklin series in the | |
| spaces $\mathscr{H}(\Delta)$ and $L^p(0,1)$ | 206 |
| CHAPTER VII. Orthogonalization and Factorization Theorems | 221 |
| §1. Orthogonalization of a system of functions by means of | |
| extension to a larger set | 222 |
| §2. Two theorems on sequences of functions | 232 |
| §3. Structure of systems with convergence in measure for l^2 | 241 |
| §4. Properties of the majorant operator for the partial sums | 244 |
| CHAPTER VIII. Theorems on the Convergence of General | |
| Orthogonal Series | 251 |
| §1. Convergence of orthogonal series almost everywhere | 251 |
| §2. Unconditional convergence almost everywhere | 265 |
| §3. Subsequences with convergence almost everywhere | 272 |
| §4. Lacunary systems | 277 |
| §5. Properties of integrated orthonormal systems | 288 |

CONTENTS vii

| CHAPTER IX. General Theorems on the Divergence of Orthogonal Series | 202 |
|---|-------|
| | 293 |
| §1. Divergence almost everywhere of rearrangements of L^2 | • • • |
| Fourier series | 293 |
| §2. Fourier coefficients of continuous functions | 301 |
| §3. Some properties of uniformly bounded orthonormal systems | 311 |
| CHAPTER X. Some Theorems on the Representation of Functions | |
| by Orthogonal Series | 341 |
| §1. Representation of functions by series that converge in | |
| measure | 342 |
| §2. Representation of functions by series that converge almost | |
| everywhere | 350 |
| §3. Two theorems on universal series | 373 |
| Appendix 1 | 385 |
| Appendix 2 | 401 |
| Notes | 421 |
| Bibliography | 435 |
| Index | 449 |