Contents

O. Introduction, Auxiliary Propositions and Notations VII

§ 1. Introduction VII

§ 2. Auxiliary Propositions and Notations XII

I. Linear Equations of Parabolic Type 1

§ 1. Analytic Semigroups 1

§ 2. The Evolution Operator U(t,s) 4

§ 3. Fractional Powers 9

9 4. Comments to Chapter I 12

II. Local Solutions of First Order Semilinear Evolution Equations 18

§ 1. Solutions of Equations with Nonlinearities Relatively Bounded to A 18

§ 2. A Nonlinear Interpolation Theorem 29

§ 3. Solutions of Equations with Nonlinearities Relatively Bounded to $A^{1-\rho}$, their Higher Regularity and the Question of Admissible Initial Data 37

§ 4. Comments to Chapter II 64

III. Local Solvability of the Equations of Navier-Stokes 67

§ 1. Solonnikov's Results for the Instationary Stokes Equation 67

§ 2. Fractional Powers of the Stokes-Operator 80

§ 3. Local Strong Solvability of the Navier-Stokes Equations 99

§ 4. Global Existence for Small Data. Extension of the Previous Results to Arbitrary Dimensions 131
§ 5. Comments to Chapter III 136

IV. Global Existence and Global Regularity for the Navier-Stokes Equations 139
§ 1. Weak Solutions 139
§ 2. Some Additional Regularity Properties for Weak Solutions in General 149
§ 3. On the Validity of the Energy Inequality and on the Regularity of the Expression $u'+\nabla\pi$ 154
§ 4. On the Uniqueness of Weak Solutions. The Connection between Weak Solutions and Local Strong Solutions 166
§ 5. Regularity of Weak Solutions. Leray's Structure Theorem 190
§ 6. Comments to Chapter IV 224

V. Global Solutions of Abstract Nonlinear Parabolic Equations and Applications 226
§ 1. Abstract Nonlinear Parabolic Equations 226
§ 2. Applications to Parabolic Systems and to the Equations of Navier-Stokes 235
§ 3. Comments to Chapter V 253

VI. References 255