CONTENTS

1.	INTRODUCTION	9
1.1.	Preliminary remarks	9
1.2.	Scope of monograph	11
1.3.	Plan of monograph, comments	12
2.	BOUNDARY VALUE PROBLEMS AND IRREGULAR NETWORKS	17
2.1.	A class of elliptic problems	17
2.2.	Irregular networks	23
2.2.1.	Networks of triangles and rectangles	23
2.2.2.	Locally irregular networks	27
2.3.	Secondary networks and boxes	29
2.3.1.	General remarks	29
2.3.2.	Secondary networks via the perpendicular bisectors (PB)	30
2.3.3.	Secondary networks via the medians (MD)	36
3.	CONSTRUCTION OF FINITE DIFFERENCE APPROXIMATIONS	40
3.1.	The principle of approximation	40
3.2.	Finite difference schemes via method (PB)	43
3.2.1.	The approximation of the balance equations	43
3.2.2.	Finite difference schemes of the type (PB)	47
3.3.	Finite difference schemes via method (MD)	48
3.3.1.	Quadrature formulas for the balance equations	48
3.3.2.	Finite difference quotients	50
3.3.3.	Finite difference schemes of the type (MD)	52
4.	ANALYTICAL AND MATRIX PROPERTIES OF THE DIFFERENCE OPERATORS A_h	55
4.1.	General remarks and notations	55
4.2.	Monotonicity and other matrix properties	57
4.2.1.	Operators A_h and matrices via method (PB)	57
4.2.2.	MD-operators A_h in comparison with PB-operators	60
4.3.	Scalar products, norms and a trace theorem	64
4.3.1.	Notations for scalar products and norms	64
4.3.2.	Ω-Ω_h-equivalence for norms of special functions	67
4.3.3.	Γ-Γ_h-relations for norms of special functions	70
4.3.4.	Relations between "continuous" and "discrete" norms	72
4.3.5.	A trace theorem and the equivalence of certain grid norms	73

4.4.	Green's formula, inequalities of Friedrichs-Poincaré-type and the positive definiteness of A_h	75
4.4.1.	Green's formula, the symmetry and the "energy" of A_h	75
4.4.2.	Inequalities of Friedrichs-Poincaré-type	78
4.4.3.	The positive definiteness of A_h	80
4.5.	A priori estimates for A_h using the W_2^1- and C-norm	85
4.5.1.	A priori estimates using the W_2^1-norm	85
4.5.2.	"Weak imbedding" of $W_2^1(\bar{\omega})$ in $C(\bar{\omega})$	88
4.5.3.	A priori estimates using the C-norm	90
5.	ERROR ESTIMATES AND CONVERGENCE	96
5.1.	Error splitting and approaches to the error estimation	96
5.1.1.	Introductory remarks and error splitting for PB-schemes	96
5.1.2.	Two kinds of error splitting for MD-schemes	99
5.1.3.	A priori estimates of the error	102
5.1.4.	Convergence for classical solutions of $C^1(\bar{\Omega})$-type $(1 \geqslant 2)$	106
5.1.5.	Convergence for generalized solutions of $W_2^1(\Omega)$-type $(1 \geqslant 2)$	108
5.2.	The error $\mathfrak{Ae}$ of the principal part of PB-operators	109
5.2.1.	The splitting of $\mathfrak{Ae}$ and first estimates	109
5.2.2.	The choice of local estimation regions	112
5.2.3.	The estimation of $\mathfrak{Ae}_k$ and $\mathfrak{Ae}_s$	114
5.2.4.	The estimation of $\mathfrak{Ae}_u$	119
5.2.5.	The weakening of the assumptions and conclusions	122
5.3.	The error $\mathfrak{Ae}$ of the principal part of MD-operators	124
5.3.1.	Splittings and estimates via centres of gravity	124
5.3.2.	Splittings and estimates via mesh midpoints	127
5.4.	The error Ψ_N for PB- and MD-schemes	132
5.4.1.	Ψ_N-estimates for grid points $x \in \omega$	132
5.4.2.	Ψ_N-estimates for grid points $x \in \gamma_{23}$	135
5.4.3.	The modification and weakening of some assumptions	142
5.5.	Convergence for $W_2^2(\Omega)$-solutions	146
5.5.1.	Convergence in the discrete W_2^1-norm	146
5.5.2.	Convergence in the discrete C-norm	149
6.	FINITE DIFFERENCE SCHEMES FOR NONSYMMETRIC PROBLEMS	150
6.1.	Construction of finite difference approximations	150
6.1.1.	Boundary value problem and balance equations	150
6.1.2.	Upwind difference quotients	152
6.1.3.	Further difference quotients and the FDSs $A_h^b y = F_h$	154
6.2.	Properties of the difference operators A_h^b	158
6.2.1.	Monotonicity and a priori estimates	158
6.2.2.	Positive definiteness and a priori estimates	161

6.3. The error ψ_K of the convection term 162
6.3.1. Error splitting and estimates 162
6.3.2. The approach to estimating ψ_K for $u \in W_2^1(\Omega)$ $(l \geq 2)$ 164
6.3.3. Local and global bounds of ψ_K 166
6.4. Convergence for $C^1(\bar{\Omega})$- and $W_2^1(\Omega)$-solutions $(l \geq 2)$ 172

7. CONCLUDING REMARKS 174

APPENDICES 181
1. Appendix DI: Relations of Differential and Integral calculus, norms 181
2. Appendix ES: Estimation of functionals on Sobolev spaces 183
3. Appendix EX: Extension of functions 185
4. Appendix GE: Some relations of geometry 186
5. Appendix IM: Imbedding and trace theorems 187
6. Appendix TR: Affine transformations of coordinates and functionals 190

REFERENCES 194

SUBJECT INDEX 203

LIST OF FIGURES 205

ABBREVIATIONS 205

NOTATIONS 206