

Contents

Preface to the Third Edition	vii
Preface to the Second Edition	ix
Preface to the First Edition	xi
CHAPTER 1	
Preliminaries on Homotopy Theory	1
1. Category Theory and Homotopy Theory	1
2. Complexes	2
3. The Spaces $\text{Map}(X, Y)$ and $\text{Map}_0(X, Y)$	4
4. Homotopy Groups of Spaces	6
5. Fibre Maps	7
PART I	
THE GENERAL THEORY OF FIBRE BUNDLES	9
CHAPTER 2	
Generalities on Bundles	11
1. Definition of Bundles and Cross Sections	11
2. Examples of Bundles and Cross Sections	12
3. Morphisms of Bundles	14
4. Products and Fibre Products	15
5. Restrictions of Bundles and Induced Bundles	17
6. Local Properties of Bundles	20
7. Prolongation of Cross Sections	21
Exercises	22

CHAPTER 3

Vector Bundles	24
1. Definition and Examples of Vector Bundles	24
2. Morphisms of Vector Bundles	26
3. Induced Vector Bundles	27
4. Homotopy Properties of Vector Bundles	28
5. Construction of Gauss Maps	31
6. Homotopies of Gauss Maps	33
7. Functorial Description of the Homotopy Classification of Vector Bundles	34
8. Kernel, Image, and Cokernel of Morphisms with Constant Rank	35
9. Riemannian and Hermitian Metrics on Vector Bundles	37
Exercises	39

CHAPTER 4

General Fibre Bundles	40
1. Bundles Defined by Transformation Groups	40
2. Definition and Examples of Principal Bundles	42
3. Categories of Principal Bundles	43
4. Induced Bundles of Principal Bundles	44
5. Definition of Fibre Bundles	45
6. Functorial Properties of Fibre Bundles	46
7. Trivial and Locally Trivial Fibre Bundles	47
8. Description of Cross Sections of a Fibre Bundle	48
9. Numerable Principal Bundles over $B \times [0, 1]$	49
10. The Cofunctor k_G	52
11. The Milnor Construction	54
12. Homotopy Classification of Numerable Principal G -Bundles	56
13. Homotopy Classification of Principal G -Bundles over CW-Complexes	58
Exercises	59

CHAPTER 5

Local Coordinate Description of Fibre Bundles	61
1. Automorphisms of Trivial Fibre Bundles	61
2. Charts and Transition Functions	62
3. Construction of Bundles with Given Transition Functions	64
4. Transition Functions and Induced Bundles	65
5. Local Representation of Vector Bundle Morphisms	66
6. Operations on Vector Bundles	67
7. Transition Functions for Bundles with Metrics	69
Exercises	71

CHAPTER 6

Change of Structure Group in Fibre Bundles	73
1. Fibre Bundles with Homogeneous Spaces as Fibres	73

2. Prolongation and Restriction of Principal Bundles	74
3. Restriction and Prolongation of Structure Group for Fibre Bundles	75
4. Local Coordinate Description of Change of Structure Group ..	76
5. Classifying Spaces and the Reduction of Structure Group	77
Exercises	77
CHAPTER 7	
The Gauge Group of a Principal Bundle	79
1. Definition of the Gauge Group	79
2. The Universal Standard Principal Bundle of the Gauge Group ..	81
3. The Standard Principal Bundle as a Universal Bundle	82
4. Abelian Gauge Groups and the Künneth Formula	83
CHAPTER 8	
Calculations Involving the Classical Groups	87
1. Stiefel Varieties and the Classical Groups	87
2. Grassmann Manifolds and the Classical Groups	90
3. Local Triviality of Projections from Stiefel Varieties	91
4. Stability of the Homotopy Groups of the Classical Groups	94
5. Vanishing of Lower Homotopy Groups of Stiefel Varieties	95
6. Universal Bundles and Classifying Spaces for the Classical Groups	95
7. Universal Vector Bundles	96
8. Description of all Locally Trivial Fibre Bundles over Suspensions	97
9. Characteristic Map of the Tangent Bundle over S^n	98
10. Homotopy Properties of Characteristic Maps	101
11. Homotopy Groups of Stiefel Varieties	103
12. Some of the Homotopy Groups of the Classical Groups	104
Exercises	107
PART II	
ELEMENTS OF K-THEORY	109
CHAPTER 9	
Stability Properties of Vector Bundles	111
1. Trivial Summands of Vector Bundles	111
2. Homotopy Classification and Whitney Sums	113
3. The K Cofunctors	114
4. Corepresentations of \tilde{K}_F	118
5. Homotopy Groups of Classical Groups and $\tilde{K}_F(S^i)$	120
Exercises	121
CHAPTER 10	
Relative K -Theory	122
1. Collapsing of Trivialized Bundles	122

2. Exact Sequences in Relative K -Theory	124
3. Products in K -Theory	128
4. The Cofunctor $L(X, A)$	129
5. The Difference Morphism	131
6. Products in $L(X, A)$	133
7. The Clutching Construction	134
8. The Cofunctor $L_n(X, A)$	136
9. Half-Exact Cofunctors	138
Exercises	139

CHAPTER 11

Bott Periodicity in the Complex Case	140
1. K -Theory Interpretation of the Periodicity Result	140
2. Complex Vector Bundles over $X \times S^2$	141
3. Analysis of Polynomial Clutching Maps	143
4. Analysis of Linear Clutching Maps	145
5. The Inverse to the Periodicity Isomorphism	148

CHAPTER 12

Clifford Algebras	151
1. Unit Tangent Vector Fields on Spheres: I	151
2. Orthogonal Multiplications	152
3. Generalities on Quadratic Forms	154
4. Clifford Algebra of a Quadratic Form	156
5. Calculations of Clifford Algebras	158
6. Clifford Modules	161
7. Tensor Products of Clifford Modules	166
8. Unit Tangent Vector Fields on Spheres: II	168
9. The Group $\text{Spin}(k)$	169
Exercises	170

CHAPTER 13

The Adams Operations and Representations	171
1. λ -Rings	171
2. The Adams ψ -Operations in λ -Ring	172
3. The γ^i Operations	175
4. Generalities on G -Modules	176
5. The Representation Ring of a Group G and Vector Bundles	177
6. Semisimplicity of G -Modules over Compact Groups	179
7. Characters and the Structure of the Group $R_F(G)$	180
8. Maximal Tori	182
9. The Representation Ring of a Torus	185

10. The ψ -Operations on $K(X)$ and $KO(X)$	186
11. The ψ -Operations on $\tilde{K}(S^n)$	187
CHAPTER 14	
Representation Rings of Classical Groups	189
1. Symmetric Functions	189
2. Maximal Tori in $SU(n)$ and $U(n)$	191
3. The Representation Rings of $SU(n)$ and $U(n)$	192
4. Maximal Tori in $Sp(n)$	193
5. Formal Identities in Polynomial Rings	194
6. The Representation Ring of $Sp(n)$	195
7. Maximal Tori and the Weyl Group of $SO(n)$	195
8. Maximal Tori and the Weyl Group of $Spin(n)$	196
9. Special Representations of $SO(n)$ and $Spin(n)$	198
10. Calculation of $RSO(n)$ and $R Spin(n)$	200
11. Relation Between Real and Complex Representation Rings	203
12. Examples of Real and Quaternionic Representations	206
13. Spinor Representations and the K -Groups of Spheres	208
CHAPTER 15	
The Hopf Invariant	210
1. K -Theory Definition of the Hopf Invariant	210
2. Algebraic Properties of the Hopf Invariant	211
3. Hopf Invariant and Bidegree	213
4. Nonexistence of Elements of Hopf Invariant 1	215
CHAPTER 16	
Vector Fields on the Sphere	217
1. Thom Spaces of Vector Bundles	217
2. S -Category	219
3. S -Duality and the Atiyah Duality Theorem	221
4. Fibre Homotopy Type	223
5. Stable Fibre Homotopy Equivalence	224
6. The Groups $J(S^k)$ and $\tilde{K}_{\text{Top}}(S^k)$	225
7. Thom Spaces and Fibre Homotopy Type	227
8. S -Duality and S -Reducibility	229
9. Nonexistence of Vector Fields and Reducibility	230
10. Nonexistence of Vector Fields and Coreducibility	232
11. Nonexistence of Vector Fields and $J(RP^n)$	233
12. Real K -Groups of Real Projective Spaces	235
13. Relation Between $KO(RP^n)$ and $J(RP^n)$	237
14. Remarks on the Adams Conjecture	240

PART III	
CHARACTERISTIC CLASSES	243
CHAPTER 17	
Chern Classes and Stiefel-Whitney Classes	245
1. The Leray-Hirsch Theorem	245
2. Definition of the Stiefel-Whitney Classes and Chern Classes	247
3. Axiomatic Properties of the Characteristic Classes	248
4. Stability Properties and Examples of Characteristic Classes	250
5. Splitting Maps and Uniqueness of Characteristic Classes	251
6. Existence of the Characteristic Classes	252
7. Fundamental Class of Sphere Bundles. Gysin Sequence	253
8. Multiplicative Property of the Euler Class	256
9. Definition of Stiefel-Whitney Classes Using the Squaring Operations of Steenrod	257
10. The Thom Isomorphism	258
11. Relations Between Real and Complex Vector Bundles	259
12. Orientability and Stiefel-Whitney Classes	260
Exercises	261
CHAPTER 18	
Differentiable Manifolds	262
1. Generalities on Manifolds	262
2. The Tangent Bundle to a Manifold	263
3. Orientation in Euclidean Spaces	266
4. Orientation of Manifolds	267
5. Duality in Manifolds	269
6. Thom Class of the Tangent Bundle	272
7. Euler Characteristic and Class of a Manifold	274
8. Wu's Formula for the Stiefel-Whitney Class of a Manifold	275
9. Stiefel-Whitney Numbers and Cobordism	276
10. Immersions and Embeddings of Manifolds	278
Exercises	279
CHAPTER 19	
Characteristic Classes and Connections	280
1. Differential Forms and de Rham Cohomology	280
2. Connections on a Vector Bundle	283
3. Invariant Polynomials in the Curvature of a Connection	285
4. Homotopy Properties of Connections and Curvature	288
5. Homotopy to the Trivial Connection and the Chern-Simons Form	290
6. The Levi-Civita or Riemannian Connection	291

CHAPTER 20

General Theory of Characteristic Classes	294
1. The Yoneda Representation Theorem	294
2. Generalities on Characteristic Classes	295
3. Complex Characteristic Classes in Dimension n	296
4. Complex Characteristic Classes	298
5. Real Characteristic Classes Mod 2	300
6. 2-Divisible Real Characteristic Classes in Dimension n	301
7. Oriented Even-Dimensional Real Characteristic Classes	304
8. Examples and Applications	306
9. Bott Periodicity and Integrality Theorems	307
10. Comparison of K -Theory and Cohomology Definitions of Hopf Invariant	309
11. The Borel-Hirzebruch Description of Characteristic Classes	309

Appendix 1

Dold's Theory of Local Properties of Bundles	312
--	-----

Appendix 2

On the Double Suspension	314
1. $H_*(\Omega S(X))$ as an Algebraic Functor of $H_*(X)$	314
2. Connectivity of the Pair $(\Omega^2 S^{2n+1}, S^{2n-1})$ Localized at p	318
3. Decomposition of Suspensions of Products and $\Omega S(X)$	319
4. Single Suspension Sequences	322
5. Mod p Hopf Invariant	326
6. Spaces Where the p th Power Is Zero	329
7. Double Suspension Sequences	333
8. Study of the Boundary Map $\Delta: \Omega^3 S^{2np+1} \rightarrow \Omega S^{2np-1}$	337

Bibliography

.....	339
-------	-----

Index

.....	348
-------	-----