Contents

Introduction 1

Chapter I : On the deterministic Malliavin calculus 17

a) Notations and assumptions 18

b) Some properties of the flows ϕ^h 20

c) Conditions of invertibility of C_1^{h,x_o} 25

d) Minimal action and the bicharacteristic flow 32

e) The split of H and the differentials of I 43

Chapter II : Brownian motion on a Riemannian manifold and the Calculus of variations 54

a) Notations and assumptions 55

b) T^*M as a quotient of T^*N 57

c) The Malliavin calculus of variations on a Riemannian manifold 60

d) The conditional process 67

e) The time reversed conditioned process 70

f) An expression for $\dfrac{\text{grad}_{x_o} p_t(x_o,y_o)}{p_t(x_o,y_o)}$ 78

g) Semi-martingale property of the conditional process on [0,t] 86

Chapter III : Conditional diffusions and conditional flows : the basic estimates 90

a) Reduction to the compact case 91

b) The basic estimates 95

c) Large deviations on flows 106

Chapter IV : Taylor expansion of the conditional probability : the elliptic case 117

a) A first split of the Hilbert space H 120

b) The split of the Brownian measure 124

c) Expression of P_1 in terms of a standard Brownian bridge 127

d) Large deviations for w^1 129

e) A local change of variables 136

f) An asymptotic expression for $p_t(x_o,y_o)$ 137

g) The Jacobian of the exponential mapping in terms of a Brownian bridge 147

h) A path integral proof of a result of Molchanov 149
i) Taylor expansion of $p_t(x_o,y_o)$ 167
j) A second split 174
k) Expansion of $p_t(x_o,x_o)$ 177

Chapter V : The hypoelliptic case : Two conjectures

a) Assumptions and notations 184
b) Remarks on the semi-martingale property 186
c) Two conjectures 188
d) The two conjectures : the elliptic case 199
e) A case study : the Heisenberg group 200

References 210