

Contents

Preface

Acknowledgements

Introduction

CHAPTER 1	EPIGRAPH-CONVERGENCE. DEFINITION. EXAMPLES	1
1.1	MODEL EXAMPLES	1
1.1.1	Homogenization of elliptic equations	1
1.1.2	Domains with many small "holes"	9
	A. Neumann boundary conditions	11
	B. Dirichlet boundary conditions. Cloud of ice	13
1.1.3	Transmission problems through thin layers	16
	A. Thin isolating layers	17
	B. Neumann's strainer	20
1.2	EPIGRAPH-CONVERGENCE. VARIATIONAL APPROACH	23
1.2.1	Variational approach. Definition in a general topological space	24
1.2.2	Epi-convergence in a first countable topological space	29
1.2.3	Epigraph-convergence: a variational notion of convergence	39
1.3	DIRECT APPROACH BY EPI-CONVERGENCE. PROOF OF EXAMPLES 1.1	41
1.3.1	Proof of epi-convergence results: various methods	41
	A. Direct method	41
	B. Compactness method	43
1.3.2	Homogenization of elliptic operators	44
1.3.3	Domains with many small holes. Neumann boundary conditions	57
	A. Homogenization of strongly connected domains	58
	B. Homogenization of thin isolating inclusions	64
	C. Homogenization of fissured elastic materials	68

1.3.4	Domains with many small holes. Dirichlet boundary conditions	70
1.3.5	Transmission problems through thin layers	82
	A. Thin isolating layer	82
	B. Neumann's strainer	87
1.4	GEOMETRIC INTERPRETATION OF EPIGRAPH CONVERGENCE	91
1.4.1	Set-convergence	92
1.4.2	Set-convergence interpretation of epi-convergence	96
1.4.3	Hypo-convergence	100
1.5	EPI-CONVERGENCE: A PARTICULAR CASE OF Γ -CONVERGENCE	102
1.5.1	Definitions of De Giorgi's Γ -limits	102
1.5.2	Epi hypo-convergence for saddle value problems	105
1.6	FURTHER EXAMPLES	106
1.6.1	Homogenization of the elastoplastic torsion of a cylindrical bar	106
1.6.2	A singular perturbation problem in optimal control theory	112
1.6.3	Epi-convergence and correctors	118
CHAPTER 2	PROPERTIES OF EPI-CONVERGENCE	122
2.1	LOWER SEMICONTINUITY OF EPI-LIMITS	122
2.2	VARIATIONAL PROPERTIES OF EPI-CONVERGENCE	127
2.3	PERTURBATION OF EPI-CONVERGENT SEQUENCES	138
2.3.1	Continuous perturbations	138
2.3.2	Examples	141
2.4	COMPACTNESS RESULTS	149
2.4.1	Setting of the problem	149
2.4.2	The abstract compactness theorem	151
2.4.3	A compactness result for a class of integral functionals	155
	A. Main theorem	155
	B. Critical cases: degeneracy of the ellipticity condition	161
	C. Critical cases: explosion of the coefficients	163

2.4.4	Compactness of the class of obstacle constraint functionals	167
A.	Examples of varying obstacle limit analysis. Thin obstacles. Fakir's bed of nails. Blow up.	167
B.	Variational inequalities with varying obstacles. General form of the limit problem	176
C.	Bilateral constraints	191
D.	Application to a problem in optimal control design	195
2.5	EPI-CONVERGENCE OF MONOTONE SEQUENCES OF FUNCTIONS	198
2.5.1	Epi-convergence of increasing sequences of functions. Penalization method	198
2.5.2	Epi-convergence of decreasing sequences of functions. Barrier and viscosity methods	205
2.6	COMPARISON OF EPI-LIMITS FOR DIFFERENT TOPOLOGIES	214
2.6.1	Epi-convergence and pointwise convergence	214
2.6.2	τ/σ -Equi-lower-semicontinuity	219
2.6.3	Example	224
2.7	MOREAU-YOSIDA PROXIMAL APPROXIMATION	226
2.7.1	Definition. Properties	226
2.7.2	Moreau-Yosida approximation and epi-convergence	232
2.8	TOPOLOGY OF EPI-CONVERGENCE	244
2.8.1	Existence of a topology inducing epi-convergence. The general situation	245
2.8.2	The locally compact case	248
2.8.3	Applications in infinite dimensional cases	259
A.	Highly oscillating potentials	259
B.	Stochastic homogenization	261
CHAPTER 3	EPI-CONVERGENCE OF CONVEX FUNCTIONS. DUALITY, MOSCO-CONVERGENCE AND CONVERGENCE OF SUBDIFFERENTIALS OPERATORS	264
3.1	CONSERVATION OF CONVEXITY BY EPI-CONVERGENCE	264
3.2	EPI-CONVERGENCE AND DUALITY	265

3.2.1	Definition and classical properties of the Fenchel transformation	265
3.2.2	Continuity properties with respect to epi-convergence of the Fenchel transformation	269
3.2.3	Examples of computation of epi-limits by duality	287
3.3	MOSCO CONVERGENCE. BICONTINUITY OF THE FENCHEL TRANSFORMATION	294
3.3.1	Introduction to Mosco-convergence via bicontinuity of the Fenchel transformation	294
3.3.2	Mosco-convergence of convex sets	298
3.4	MOSCO-CONVERGENCE AND CONVERGENCE OF MOREAU-YOSIDA APPROXIMATES	300
3.4.1	Differentiability properties of Moreau-Yosida approximates	300
3.4.2	Equivalence between Mosco-convergence and pointwise convergence of Moreau-Yosida approximates	305
3.4.3	Geometric interpretation of Mosco-convergence of convex sets	322
3.5	TOPOLOGY OF MOSCO-CONVERGENCE	323
3.5.1	Definition and properties of Mosco-convergence topology	323
3.5.2	Dense subsets of $\Gamma_M(X)$ and approximation of closed convex functions	328
3.6	APPLICATIONS OF MOSCO-CONVERGENCE	336
3.6.1	Convergence problems in optimal control	336
3.6.2	Stability results for variational inequalities with obstacle constraints	347
3.7	MAXIMAL MONOTONE OPERATORS. GRAPH CONVERGENCE	351
3.7.1	Yosida approximation of maximal monotone operators	352
3.7.2	Graph convergence of maximal monotone operators	360
3.7.3	Topology of resolvent (or graph) convergence on the class of maximal monotone operators	364

3.8	CONVERGENCE OF SUBDIFFERENTIAL OPERATORS	371
3.8.1	The class of subdifferential operators is closed for G-convergence	371
3.8.2	Equivalence between G-convergence of $\{\partial F^n; n \in \mathbb{N}\}$ and Mosco convergence of $\{F^n; n \in \mathbb{N}\}$	372
3.8.3	Epi-convergence for the weak topology of functions and convergence of subdifferentials	376
3.9	CONVERGENCE OF OPERATORS. EXAMPLES	378
3.9.1	G-convergence of elliptic operators. Convergence of the spectrum	378
3.9.2	Convergence of semi-groups	386
	BIBLIOGRAPHY	390