

# C O N T E N T S

Notations	9
<b>1. FOUNDATIONS</b>	<b>10</b>
1.1. Asymptotic approximations and asymptotic expansions	10
1.2. Expansion operators	14
1.3. Asymptotic solutions	22
1.4. Stability of asymptotic methods	28
1.5. Significant degeneration of differential operators	31
Comments to Chapter 1	35
<b>2. ESTIMATES OF THE SOLUTIONS OF ELLIPTIC AND PARABOLIC BOUNDARY VALUE PROBLEMS</b>	<b>37</b>
2.1. Notations and Definitions	37
2.2. Existence and a priori estimates of classical solutions	39
2.3. A-priori estimates for singularly perturbed elliptic problems	45
2.4. A-priori estimates for singularly perturbed parabolic problems	51
2.5. Application of Banach's fixed point theorem. Estimates of the solutions of special elliptic problems of second order	53
Comments to Chapter 2	56
<b>3. PROPERTIES OF SOLUTIONS OF GLOBAL PROBLEMS FOR ELLIPTIC AND PARABOLIC BOUNDARY VALUE PROBLEMS</b>	<b>57</b>
Comments to Chapter 3	68
<b>4. ASYMPTOTIC SOLUTIONS OF ELLIPTIC AND PARABOLIC BOUNDARY VALUE PROBLEMS</b>	<b>69</b>
4.1. Global asymptotic solutions for equations of second order	69
4.2. Asymptotic solutions for equations of second order in the simplest case	80
4.2.1. Uniform asymptotic solutions in the case $\Gamma = \Gamma_+$	80
4.2.2. Uniform asymptotic solutions in the case $\Gamma = \square$	85
4.2.3. Uniform asymptotic solutions in the case $\Gamma = \square_0$	86
4.2.4. Remarks on multiply connected domains	89
4.3. Asymptotic solutions for equations of arbitrary order	90
4.3.1. Uniform asymptotic solutions in the case $\Gamma = \Gamma_+$	90
4.3.2. Uniform asymptotic solutions in the case $\Gamma = \square$	93
4.3.3. Uniform asymptotic solutions in the case $\Gamma = \square_0$	97
4.4. Tangential characteristics	99
4.4.1. The elliptic case	99
4.4.2. The parabolic case	107

4.5. Turning points	111
4.5.1. Isolated turning points	113
4.5.1.1. An instable singular point in the case $\Gamma = \Gamma_+$	113
4.5.1.2. An instable singular point in the case $\Gamma = \Gamma_0$	117
4.5.1.3. Closed characteristics in the neighbourhood of the boundary	122
4.5.1.4. An absolute stable singular point with $\Gamma = \Gamma_-$	124
4.5.2. Singular lines	126
4.5.2.1. A singular line parallel to the characteristics	127
4.5.2.2. A singular line perpendicular to the characteristics	128
4.6. Nonlinear elliptic problems of second order	135
4.6.1. Problems without boundary layers	135
4.6.2. Problems with boundary layers	137
Comments to Chapter 4	144
Appendix: Boundary layer problems	148
A 1 Ordinary boundary layers	148
1.1. Constant coefficients (1)	148
1.2. Constant coefficients (2)	149
1.3. Variable coefficients (1)	149
1.4. Variable coefficients (2)	150
A 2 Parabolic boundary layers (I)	151
2.1. The simplest case of an equation of second order	151
2.2. The simplest case of an equation of arbitrary order	152
2.3. Periodicity in an additional independent variable	154
2.4. A more general equation of second order	155
A 3 Parabolic boundary layers (II)	156
3.1. Periodic solutions of equations of second order in the simplest case	156
3.2. Periodic solutions for an equation of arbitrary order	158
3.3. A more general equation of second order	160
3.4. A special equation of second order	161
A 4 Parabolic boundary layers (III)	162
4.1. A degenerating equation of second order (1)	162
4.2. A degenerating equation of second order (2)	164
BIBLIOGRAPHY	165
SUBJECT INDEX	175