
Contents

1 INTRODUCTION / 1

LESSON 1 Introduction to Partial Differential Equations / 3

2 DIFFUSION-TYPE PROBLEMS / 9

LESSON 2 Diffusion-Type Problems (Parabolic Equations) / 11

LESSON 3 Boundary Conditions for Diffusion-Type Problems / 19

LESSON 4 Derivation of the Heat Equation / 27

LESSON 5 Separation of Variables / 33

LESSON 6 Transforming Nonhomogeneous BCs into Homogeneous Ones / 43

LESSON 7 Solving More Complicated Problems by Separation of Variables / 49

LESSON 8 Transforming Hard Equations into Easier Ones / 58

LESSON 9 Solving Nonhomogeneous PDEs (Eigenfunction Expansions) / 64

LESSON 10 Integral Transforms (Sine and Cosine Transforms) / 72

LESSON 11 The Fourier Series and Transform / 81

LESSON 12 The Fourier Transform and its Application to PDEs / 89

LESSON 13 The Laplace Transform / 97

LESSON 14 Duhamel's Principle / 106

LESSON 15 The Convection Term u_x in Diffusion Problems / 112

3 HYPERBOLIC-TYPE PROBLEMS / 121

LESSON 16 The One Dimensional Wave Equation (Hyperbolic Equations) / 123

LESSON 17 The D'Alembert Solution of the Wave Equation / 129

LESSON 18 More on the D'Alembert Solution / 137

LESSON 19 Boundary Conditions Associated with the Wave Equation / 146

LESSON 20 The Finite Vibrating String (Standing Waves) / 153

LESSON 21	The Vibrating Beam (Fourth-Order PDE)	/	161
LESSON 22	Dimensionless Problems	/	168
LESSON 23	Classification of PDEs (Canonical Form of the Hyperbolic Equation)	/	174
LESSON 24	The Wave Equation in Two and Three Dimensions (Free Space)	/	183
LESSON 25	The Finite Fourier Transforms (Sine and Cosine Transforms)	/	191
LESSON 26	Superposition (The Backbone of Linear Systems)	/	198
LESSON 27	First-Order Equations (Method of Characteristics)	/	205
LESSON 28	Nonlinear First-Order Equations (Conservation Equations)	/	213
LESSON 29	Systems of PDEs	/	223
LESSON 30	The Vibrating Drumhead (Wave Equation in Polar Coordinates)	/	232

4 ELLIPTIC-TYPE PROBLEMS / 243

LESSON 31	The Laplacian (an intuitive description)	/	245
LESSON 32	General Nature of Boundary-Value Problems	/	253
LESSON 33	Interior Dirichlet Problem for a Circle	/	262
LESSON 34	The Dirichlet Problem in an Annulus	/	270
LESSON 35	Laplace's Equation in Spherical Coordinates (Spherical Harmonics)	/	280
LESSON 36	A Nonhomogeneous Dirichlet Problem (Green's Functions)	/	290

5 NUMERICAL AND APPROXIMATE METHODS / 299

LESSON 37	Numerical Solutions (Elliptic Problems)	/	301
LESSON 38	An Explicit Finite-Difference Method	/	309
LESSON 39	An Implicit Finite-Difference Method (Crank-Nicolson Method)	/	317
LESSON 40	Analytic versus Numerical Solutions	/	324
LESSON 41	Classification of PDEs (Parabolic and Elliptic Equations)	/	331
LESSON 42	Monte Carlo Methods (An Introduction)	/	340
LESSON 43	Monte Carlo Solutions of Partial Differential Equations	/	346
LESSON 44	Calculus of Variations (Euler-Lagrange Equations)	/	353

LESSON 45 Variational Methods for Solving PDEs (Method of Ritz) / 362

LESSON 46 Perturbation Method for Solving PDEs / 370

LESSON 47 Conformal-Mapping Solution of PDEs / 379

APP. 1 INTEGRAL TRANSFORM TABLES / 389

APP. 2 PDE CROSSWORD PUZZLE / 398

INDEX / 401