

CONTENTS

| | |
|---|-----------|
| 1. Linear Differential Equations | 1 |
| 1.1. Linear Operators, 1 | |
| 1.2. Ordinary Differential Equations, 2 | |
| 1.3. Homogeneous Linear ODE with Constant Coefficients, 5 | |
| 1.4. Euler's ODE, 7 | |
| 1.5. Series Solutions, 8 | |
| 1.6. Frobenius Method, 13 | |
| 1.7. Numerical Solutions, 19 | |
| 1.8. Linear PDEs, 23 | |
| 1.9. Classification of a Linear PDE of Second Order, 23 | |
| 1.10. Boundary Value Problems with PDEs, 24 | |
| 1.11. Second Order Linear PDEs with Constant Coefficients, 26 | |
| 1.12. Separation of Variables, 35 | |
| | |
| 2. Orthogonal Sets of Functions | 40 |
| 2.1. Orthogonality and Vectors, 40 | |
| 2.2. Orthogonal Functions, 42 | |
| 2.3. Complex Functions, 46 | |
| 2.4. Additional Concepts of Orthogonality, 47 | |
| 2.5. The Sturm–Liouville Boundary Value Problem, 50 | |
| 2.6. Uniform Convergence of Series, 58 | |
| 2.7. Series of Orthogonal Functions, 61 | |
| 2.8. Approximation by Least Squares, 64 | |
| 2.9. Completeness of Sets, 65 | |
| | |
| 3. Fourier Series | 68 |
| 3.1. Piecewise Continuous Functions, 68 | |
| 3.2. A Basic Fourier Series, 72 | |
| 3.3. Even and Odd Functions, 76 | |

| | | |
|-----------|--|------------|
| 3.4. | Fourier Sine and Cosine Series, 77 | |
| 3.5. | Complex Fourier Series, 80 | |
| 3.6. | Harmonic Analysis, 82 | |
| 3.7. | Uniform Convergence of Fourier Series, 89 | |
| 3.8. | Differentiation of Fourier Series, 92 | |
| 3.9. | Integration of Fourier Series, 94 | |
| 3.10. | Double Fourier Series, 97 | |
| 4. | Fourier Integrals | 102 |
| 4.1. | Uniform Convergence of Integrals, 102 | |
| 4.2. | A Generalization of the Fourier Series, 107 | |
| 4.3. | Fourier Sine and Cosine Integrals, 109 | |
| 4.4. | The Exponential Fourier Integral, 112 | |
| 5. | Bessel Functions | 117 |
| 5.1. | The Gamma Function and the Bessel Function, 117 | |
| 5.2. | Additional Bessel Functions, 120 | |
| 5.3. | Differential Equations Solvable with Bessel Functions, 122 | |
| 5.4. | Special Bessel Functions and Identities, 124 | |
| 5.5. | An Integral Form for $J_n(x)$, 130 | |
| 5.6. | Singular SLPs, 133 | |
| 5.7. | Orthogonality of Bessel Functions, 134 | |
| 5.8. | Orthogonal Series of Bessel Functions, 137 | |
| 5.9. | Bessel Functions and Cylindrical Geometry, 140 | |
| 6. | Legendre Polynomials | 142 |
| 6.1. | Solutions to the Legendre Equation, 142 | |
| 6.2. | Rodrigues' Formula for Legendre Polynomials, 146 | |
| 6.3. | A Generating Function for $P_n(x)$, 149 | |
| 6.4. | The Legendre Polynomial $P_n(\cos \theta)$, 151 | |
| 6.5. | Orthogonality and Norms of $P_n(x)$, 152 | |
| 6.6. | Legendre Series, 154 | |
| 6.7. | Legendre Polynomials and Spherical Geometry, 158 | |
| 6.8. | Spherical Harmonics, 161 | |
| 6.9. | The Generalized Legendre Equation, 162 | |
| 7. | Integral Transforms | 168 |
| 7.1. | Laplace Transforms, 168 | |
| 7.2. | Existence of the Transform, 169 | |
| 7.3. | The Gamma Function and Laplace Transforms, 170 | |
| 7.4. | Transforms of Derivatives, 172 | |
| 7.5. | Derivatives of Transforms, 172 | |
| 7.6. | The Inverse Laplace Transform, 173 | |

- 7.7. Solutions of ODEs and IVPs, 173
- 7.8. Partial Fractions, 174
- 7.9. The Unit Step Function, 175
- 7.10. Shifting Properties, 176
- 7.11. The Dirac Delta Function, 177
- 7.12. Convolution, 180
- 7.13. Laplace Transform Method for PDEs, 186
- 7.14. Finite Fourier Transforms, 189
- 7.15. Fourier Transforms, 191
- 7.16. The Discrete Fourier Transform, 197
- 7.17. The Fast Fourier Transform, 203
- 7.18. Fourier Transforms of Functions of Two Variables, 208
- 7.19. Hankel Transforms, 209
- 7.20. Legendre Transform, 214
- 7.21. Mellin Transform, 215

8. Application of BVPs

219

- 8.1. The Vibrating String, 219
- 8.2. Verification and Uniqueness of the Solution of the Vibrating String Problem, 225
- 8.3. The Vibrating String with Two Nonhomogeneous Conditions, 228
- 8.4. Longitudinal Vibrations along an Elastic Rod, 230
- 8.5. Heat Conduction, 236
- 8.6. Numerical Solution of the Heat Equation, 241
- 8.7. Verification and Uniqueness of the Solution for the Heat Problem, 242
- 8.8. Gravitational Potential, 246
- 8.9. Laplace's Equation, 247
- 8.10. Numerical Solution of the Laplace Equation, 251
- 8.11. Temperature in a Circular Disk with Insulated Faces, 254
- 8.12. Steady State Temperature in a Right Semicircular Cylinder, 256
- 8.13. Harmonic Interior of a Right Circular Cylinder, 260
- 8.14. Steady State Temperature Distribution in a Sphere, 264
- 8.15. Potential for a Sphere, 267

9. Additional Applications

270

- 9.1. Mechanical and Electrical Oscillations, 270
- 9.2. The Vibrating Membrane, 273
- 9.3. Vibrations of a Circular Membrane Dependent on Distance from Center, 280
- 9.4. The Vibrating String with an External Force, 283
- 9.5. Nonhomogeneous End Temperatures in a Rod, 289
- 9.6. A Rod with Insulated Ends, 291

| | |
|---|------------|
| 9.7. A Semi-Infinite Bar, 295 | |
| 9.8. An Infinite Bar, 297 | |
| 9.9. Discrete Fourier Transform Solutions, 305 | |
| 9.10. A Semi-Infinite String, 307 | |
| 9.11. A Semi-Infinite String with Initial Velocity, 310 | |
| References | 315 |
| Answers to Exercises | 317 |
| Appendix 1 Selected Integrals | 340 |
| Appendix 2 Table of Laplace Transforms | 342 |
| Appendix 3 Tables of Finite Fourier Transforms | 344 |
| Appendix 4 Tables of Fourier Transforms | 346 |
| Index | 349 |