## Contents

Preface	
Introduction	1
Chapter I. The Origins of Free and Moving Boundary Problems	4
1. FREE AND MOVING BOUNDARY PROBLEMS INDUCED BY THE PHYSICAL INACCEPTABILITY OF SOLUTIONS OF THE UNDERLYING DIFFERENTIAL EQUATIONS: SHOCK WAVES	4
<ol><li>FREE AND MOVING BOUNDARY PROBLEMS INDUCED BY A SUDDEN CHANGE IN PHYSICAL PROPERTIES</li></ol>	6
A. Frictional and Contact Problems in Solid Mechanics B. Problems in Heat Conduction C. Problems in Molecular Diffusion D. Problems in Incompressible Hydrodynamics (a) Inviscid Flow (b) Slow Flow E. Problems in Porous Media	7 13 16 18 18 20 24
3. FURTHER REMARKS	26
Chapter II. Intuitive Ideas about Weak and Variational Methods  1. REFORMULATION OF FREE AND MOVING BOUNDARY PROBLEMS  (i) Integral Equation Methods  (ii) Mapping Methods  (iii) Discretisations  (iv) Embedding and Fixed Domain Methods	29 30 30 33 36 37
2. WEAK FORMULATION OF FREE AND MOVING BOUNDARY PROBLEMS  (i) Problems of Type 1 (p. 4)  (ii) Problems of Type 2 (p. 6)  (a) The Porous Medium Equation  (b) Stefan Problems  (c) Other Problems in Heat and Mass Transfer  (d) Frictional Oscillations	39 39 41 41 43 47 48
3. VARIATIONAL METHODS FOR FREE AND MOVING BOUNDARY PROBLEMS  (a) Vortex Flows and Plasma Confinement  (b) Obstacle Problems  (c) Elastic-Plastic Torsion Problem  (d) Parabolic Problems  (e) Problems with Discontinuous Derivatives at the Unknown Boundary	49 49 50 53 54 55
4. SUMMARY	59
Chapter III. The Enthalpy Method for the Stefan Problem	61

1. WEAK FORMULATION OF THE STEFAN PROBLEM

61

2. NUMERICAL ANALYSIS OF THE ENTHALPY METHOD	68
<ul><li>(i) Discretisation of the Weak Formulation</li><li>(ii) Existence and Uniqueness of the Solution to the Discrete</li></ul>	68
Problem	74 77
<pre>(iii) A Successive Overrelaxation Algorithm (iv) Stability</pre>	7 <i>7</i>
<ul><li>(v) Convergence of the Finite Element Discretisation</li></ul>	80 83
(vi) Numerical Examples	83
Chapter IV. Elliptic and Parabolic Variational Inequalities	92
1. THE OBSTACLE PROBLEM FOR A MEMBRANE	92
2. VARIATIONAL INEQUALITIES	95
3. VARIATIONAL INEQUALITY FORMULATION OF THE OBSTACLE PROBLEM	101
4. THE DAM PROBLEM	105
5. NUMERICAL ANALYSIS OF ELLIPTIC VARIATIONAL INEQUALITIES	107
<ul><li>(i) Discretisations</li><li>(ii) Error Estimates</li></ul>	107 109
(iii) Methods for Solving Linear Complementarity Problems	111
6. NUMERICAL EXAMPLES	117
7. TIME DEPENDENT PROBLEMS	127
(i) The Oxygen Consumption Problem	127 129
(ii) The One-phase Stefan Solidification Problem	132
8. PARABOLIC VARIATIONAL INEQUALITIES	137
<ol> <li>NUMERICAL SOLUTION OF PARABOLIC VARIATIONAL INEQUALITIES</li> <li>(i) Finite difference approximations</li> </ol>	137
(ii) A Finite Element Approximation	138
(iii) Stability (iv) Convergence	140 142
(v) Numerical Examples	146
Chapter V. Other Applications	152
A. Contact Problems	152
B. Generalised Stefan Problems	155
C. Irrotational Water Waves	156
D. Lubrication Theory	158 160
E. Immiscible Fluid Flow in Porous Media and Hele-Shaw Cells F. Stability and Mushy Regions	161
Chapter VI. Generalisations and Conclusions	170
1. CONSERVATION LAWS AND FREE AND MOVING BOUNDARY PROBLEMS	170
<ol> <li>THE DERIVATION OF VARIATIONAL INEQUALITIES FROM A CLASS OF CONSERVATION LAWS</li> </ol>	174
3. CONCLUSION	181
REFERENCES	183
APPENDIX	201
INDEX	210