

Contents

PREFACE	vii
1. Analysis of Vectors and Matrices	
1.1 Norms of Vectors and Matrices	1
1.2 Canonical Forms of Matrices	3
1.3 Sequences and Series of Matrices	6
1.4 Logarithms of Matrices	10
1.5 Differentiation and Integration of Matrices	15
2. Basic Theorems Concerning Ordinary Differential Equations	
2.1 Fundamental Existence Theorems	17
2.2 Dependence of the Solution on Initial Conditions and Parameters	19
3. Linear Differential Systems	
3.1 Linear Homogeneous Systems	24
3.2 Linear Nonhomogeneous Systems	31
4. Orbits of Autonomous Systems	
4.1 Introduction	34
4.2 Critical Points of Autonomous Systems	37
4.3 Periodic Solutions and Closed Orbits	38
4.4 Continuity of Orbits	39
5. Moving Orthonormal Systems along a Closed Orbit	
5.1 A Method to Construct a Moving Orthonormal System along a Closed Orbit	42
5.2 Equations of Orbits with Respect to the Moving Orthogonal System	48
5.3 Multipliers of Solutions of the Normal Variation Equation	51

6. Stability

6.1	Definition of Stability.	54
6.2	Fundamental Theorems Concerning Stability.	56
6.3	Orbital Stability	80
6.4	Orbital Stability of Critical Points	82
6.5	Orbital Stability of Periodic Solutions.	87

7. Perturbation of Autonomous Systems

7.1	Fundamental Formulas	91
7.2	Periodic Solutions of the Perturbed System	93
7.3	Stability of the Periodic Solution of the Perturbed System	97

8. Perturbation of Fully Oscillatory Systems

8.1	Universal Periods.	102
8.2	Preliminary Theorem	106
8.3	Perturbation of a Fully Oscillatory System.	113
8.4	An Example	115

9. Perturbation of Partially Oscillatory Systems

9.1	The Reduced Form of the Partially Oscillatory System	120
9.2	Perturbation of a Partially Oscillatory System	139
9.3	Stability of the Periodic Solution of the Perturbed System	152
9.4	An Example	159

10. Analysis of Two-Dimensional Autonomous Systems

10.1	Fundamental Formulas	163
10.2	Stability of a Periodic Solution of the Unperturbed System	166
10.3	Perturbation.	167
10.4	Perturbation of a Fully Oscillatory System.	167
10.5	Fundamental Formulas for Analytic Systems.	179
10.6	Stability of a Periodic Solution of the Analytic Unperturbed System	180
10.7	Perturbation of Analytic Systems.	185
10.8	Multiplicities of Closed Orbits	195
10.9	Perturbation of Analytic Fully Oscillatory Systems	199

11. Numerical Computation of Periodic Solutions	
11.1 A Method to Compute a Periodic Solution	204
11.2 The Two-Dimensional Case	213
11.3 Periodic Solutions of the Autonomous van der Pol Equation.	216
11.4 Remarks	225
12. Center of the Autonomous System	
12.1 First Reduction	229
12.2 The Universal Period of the Orbit	234
12.3 The Canonical Form of the Autonomous System in the Neighborhood of the Center.	239
13. Inverse Problems Connected with Periods of Oscillations Described by $\ddot{x} + g(x) = 0$	
13.1 Preliminaries.	244
13.2 Lemmas.	246
13.3 The Period Function Associated with the Maximum Velocity.	259
13.4 The Period Functions Associated with the Amplitude and the Half-Amplitudes	270
Appendix. The Newton Method and Step-by-Step Methods for Ordinary Differential Equations	
A.1 The Iterative Method to Solve Equations Numerically	281
A.2 The Newton Method	288
A.3 A Numerical Illustration of the Newton Method	299
A.4 Step-by-Step Methods for Ordinary Differential Equations.	303
A.5 Convergence of the Runge-Kutta Method	309
A.6 Convergence of the Multi-Step Methods.	313
BIBLIOGRAPHY	323
SUBJECT INDEX	327