## TABLE OF CONTENTS

PREFACE	xiii
PREFACE BY SECOND AUTHOR	xvii
CHAPTER I / MONOTONE CONVERGENCE AND POSITIVE OPERATORS  1. Introduction 2. Monotone Operators 3. Monotonicity 4. Convergence 5. Differential Equations with Initial Conditions 6. Two-Point Boundary Conditions 7. Nonlinear Heat Equation 8. The Nonlinear Potential Equation Bibliography and Comments	1 1 1 2 2 2 3 3 3 4
CHAPTER II / CONSERVATION  1. Introduction  2. Analytic and Physical Preliminaries  3. The Defining Equations  4. Limiting Differential Equations  5. Conservation for the Discrete Approximation  6. Existence of Solutions for Discrete Approximation  7. Conservation for Nonlinear Equations  8. The Matrix Riccati Equation  9. Steady-State Neutron Transport with Discrete Energy Levels  10. Analytic Preliminaries  11. Reflections, Transmission, and Loss Matrices  12. Existence and Uniqueness of Solutions  13. Proof of Conservation Relation  14. Proof of Nonnegativity  15. Statement of Result Bibliography and Comments	5 5 5 9 11 11 12 16 16 17 18 19 22 23 24 26 26
CHAPTER III / DYNAMIC PROGRAMMING AND PARTIAL DIFFERENTIAL EQUATIONS 1. Introduction 2. Calculus of Variations as a Multistage Decision	28 28
Process	28

<ul><li>3. A New Formalism</li><li>4. Layered Functionals</li><li>5. Dynamic Programming Approach</li><li>6. Quadratic Case</li><li>7. Bounds</li><li>Bibliography and Comments</li></ul>	29 31 32 34 34 34
CHAPTER IV / THE EULER-LAGRANGE EQUATIONS AND	
CHARACTERISTICS	36
1. Introduction	36
<ul><li>2. Preliminaries</li><li>3. The Fundamental Relations of the Calculus of</li></ul>	37
Variations	38
4. The Variational Equations	39
5. The Eulerian Description	43
6. The Lagrangian Description	47
7. The Hamiltonian Description	52
8. Characteristics	53
Bibliography and Comments	58
CHAPTER V / QUASILINEARIZATION AND A NEW METHOD OF SUCCESSIVE APPROXIMATIONS	59
1. Introduction	59
2. The Fundamental Variational Relation	59
3. Successive Approximations	60
4. Convergence	60 61
Bibliography and Comments	01
CHAPTER VI / THE VARIATION OF CHARACTERISTIC VALUE AND FUNCTIONS	s 62
1. Introduction	62
2. Variational Problem	63
3. Dynamic Programming Approach	65
4. Variation of the Green's Function	66
5. Justification of Equating Coefficients	69
6. Change of Variable	69
7. Analytic Continuation	71
8. Analytic Character of Green's Function 9. Alternate Derivation of Expression for $\phi(x)$	72 73
10. Variation of Characteristic Values and	73
Characteristic Functions	74
11. Matrix Case	76
12. Integral Equations	80
Bibliography and Comments	81

TABLE OF CONTENTS

ix

CHAPTER VII / THE HADAMARD VARIATIONAL FORMULA  1. Introduction 2. Preliminaries 3. A Minimum Problem 4. A Functional Equation 5. The Hadamard Variation 6. Laplace-Beltrami Operator 7. Inhomogeneous Operator Bibliography and Comments	82 82 83 85 86 86 86
CHAPTER VIII / THE TWO-DIMENSIONAL POTENTIAL EQUATION  1. Introduction 2. The Euler-Lagrange Equation 3. Inhomogeneous and Nonlinear Cases 4. Green's Function 5. Two-Dimensional Case 6. Discretization 7. Rectangular Region 8. Associated Minimization Problem 9. Approximation from Above 10. Discussion 11. Semidiscretization 12. Solution of the Difference Equations 13. The Potential Equation 14. Discretization 15. Matrix-Vector Formulation 16. Dynamic Programming 17. Recurrence Equations 18. The Calculations 19. Irregular Regions Bibliography and Comments	88 88 88 89 90 91 91 93 93 93 94 95 96 97 98 100 102 102
CHAPTER IX / THE THREE-DIMENSIONAL POTENTIAL EQUATION  1. Introduction  2. Discrete Variational Problems  3. Dynamic Programming  4. Boundary Conditions  5. Recurrence Relations  6. General Regions  7. Discussion  Bibliography and Comments	N 103 103 103 105 107 107 108 109
CHAPTER X / THE HEAT EQUATION  1. Introduction  2. The One-Dimensional Heat Equation	110 110 110

4. Some Numerical Results 5. Multidimensional Case Bibliography and Comments	113 118 119
CHAPTER XI / NONLINEAR PARABOLIC EQUATIONS  1. Introduction 2. Linear Equation 3. The Non-negativity of the Kernel 4. Monotonicity of Mean Values 5. Positivity of the Parabolic Operator 6. Nonlinear Equations 7. Asymptotic Behavior 8. Extensions Bibliography and Comments	120 120 120 121 121 123 125 126 126 127
CHAPTER XII / DIFFERENTIAL QUADRATURE  1. Introduction 2. Differential Quadrature 3. Determination of Weighting Coefficients 4. Numerical Results for First Order Problems 5. Systems of Nonlinear Partial Differential Equations 6. Higher Order Problems 7. Error Representation 8. Hodgkin-Huxley Equation 9. Equations of the Mathematical Model 10. Numerical Method 11. Conclusion Bibliography and Comments	129 129 130 132 135 138 140 141 143 145 146
CHAPTER XIII / ADAPTIVE GRIDS AND NONLINEAR EQUATIONS  1. Introduction 2. The Equation u <sub>t</sub> = -uu <sub>x</sub> 3. An Example 4. Discussion 5. Extension 6. Higher Order Approximations Bibliography and Comments	148 148 149 150 150 151 152
CHAPTER XIV / INFINITE SYSTEMS OF DIFFERENTIAL EQUATIONS 1. Introduction 2. Burgers' Equation 3. Some Numerical Examples 4. Two-Dimensional Case 5. Closure Techniques 6. A Direct Method	153 153 154 156 160 162 163

TABLE OF CONTENTS

7.	Extrapolation	163
	Difference Approximations	166
	An Approximating Algorithm	166
	Numerical Results	167
	Higher Order Approximation	170
	Truncation	170
13.	Associated Equation	172
14.	Discussion of Convergence of u(N)	173
15.	The Fejer Sum	173
	The Modified Truncation	174
	liography and Comments	175
СНУБЩ	ER XV / GREEN'S FUNCTIONS	176
	Introduction	176
	The Concept of the Green's Function	177
	Sturm-Liouville Operator	180
	Properties of the Green's Function for the	100
→•	Sturm-Liouville Equation	181
5	Properties of the $\delta$ Function	187
	Distributions	188
	Symbolic Functions	189
	Derivative of Symbolic Functions	189
	What Space Are We Considering?	194
		197
	Boundary Conditions Properties of Operator L	203
		203
	Adjoint Operators	207
	n-th Order Operators	201
⊥4•	Boundary Conditions for the Sturm-Liouville	208
7.5	Equation	209
	Green's Function for Sturm-Liouville Operator	210
	Solution of the Inhomogeneous Equation	213
	Solving Non-Homogeneous Boundary Conditions	213
18.	Boundary Conditions Specified on Finite	015
	Interval [a, b]	215
19.	Scalar Products	220
20.	Use of Green's Function to Solve a Second-order	000
	Stochastic Differential Equation	222
	Use of Green's Function in Quantum Physics	226
	Use of Green's Functions in Transmission Lines	226
23.	Two-Point Green's Functions - Generalization to	
	n-point Green's Functions	22
24.	Evaluation of Arbitrary Functions for	
	Nonhomogeneous Boundary Conditions by Matrix	
	Equations	230
25.	Mixed Boundary Conditions	23

хi

26. Some General Properties	232
1. Nonnegativity of Green's Functions and Solutions	232
2. Variation-Diminishing Properties of Green's	
<u>Functions</u>	233
Notes	235
Bibliography	235
HAPTER XVI / APPROXIMATE CALCULATION OF GREEN'S FUNCTIONS	237
HAPTER XVII / GREEN'S FUNCTIONS FOR PARTIAL DIFFERENTIAL EQUATIONS	243
1. Introduction	243
<ol><li>Green's Functions for Multidimensional Problems in Cartesian Coordinates</li></ol>	243
3. Green's Functions in Curvilinear Coordinates	244
4. Properties of $\delta$ Functions for Multi-dimensional Case	246
HAPTER XVIII / THE ITÔ EQUATION AND A GENERAL STOCHASTIC MODEL FOR DYNAMICAL SYSTEMS Bibliography	248 252
MADEED MAY / NONLEMEND DADEELS DESERVED AT BOLLASTONG	
HAPTER XIX / NONLINEAR PARTIAL DIFFERENTIAL EQUATIONS AND THE DECOMPOSITION METHOD	254
	262
1. Parametrization and the ${ t A}_{ t D}$ Polynomials	
2. Inverses for Non-simple Differential Operators	274
3. Multidimensional Green's Functions by Decomposition	275
Method	21)
4. Relationships Between Green's Functions and the	
Decomposition Method for Partial Differential	277
Equations 5 Compand of Strategy	281
<ol> <li>Separable Systems</li> <li>The partitioning Method of Butkovsky</li> </ol>	281
7. Computation of the $A_n$	282
**	
8. The Question of Convergence	284
Bibliography	287
	000
NDEX	289