

# *Contents*

|   |    |
|---|----|
| Preface . . . . .   | xi |
| <b>Part A: Theory</b>   |    |
| Chapter 1. Introduction . . . . .   | 3  |
| 1.1. Classification of Volterra equations . . . . .                             | 3  |
| 1.2. Connection between Volterra equations and initial value problems . . . . . | 7  |
| Notes on Chapter 1 . . . . .  | 10 |
| Chapter 2. Some Applications of Volterra Equations . . . . .                    | 13 |
| 2.1. History-dependent problems . . . . .                                       | 14 |
| 2.2. Applications in systems theory . . . . .                                   | 15 |
| 2.3. Problems in heat conduction and diffusion . . . . .                        | 19 |
| 2.4. Some problems in experimental inference . . . . .                          | 23 |
| Notes on Chapter 2 . . . . .  | 26 |
| Chapter 3. Linear Volterra Equations of the Second Kind . . . . .               | 29 |
| 3.1. Existence and uniqueness of the solution . . . . .                         | 29 |
| 3.2. The resolvent kernel . . . . .   | 35 |
| 3.3. Some qualitative properties of the solution . . . . .                      | 38 |
| 3.4. Systems of equations . . . . .   | 46 |
| 3.5. Equations with unbounded kernels . . . . .                                 | 47 |
| 3.6. Integrodifferential equations . . . . .                                    | 49 |
| Notes on Chapter 3 . . . . .  | 50 |
| Chapter 4. Nonlinear Equations of the Second Kind . . . . .                     | 51 |
| 4.1. Successive approximations for Lipschitz continuous kernels . . . . .       | 51 |
| 4.2. Existence and uniqueness for more general kernels . . . . .                | 54 |

|   |        |
|---|--------|
| 4.3. Properties of the solution . . . . .   | 58     |
| 4.4. Unbounded kernels and systems of equations . . . . .                         | 62     |
| 4.5. The resolvent equation . . . . .   | 63     |
| Notes on Chapter 4 . . . . .  | 65     |
| <br>Chapter 5. Equations of the First Kind . . . . .                              | <br>67 |
| 5.1. Equations with smooth kernels . . . . .                                      | 67     |
| 5.2. Abel equations . . . . .   | 71     |
| Notes on Chapter 5 . . . . .  | 76     |
| <br>Chapter 6. Convolution Equations . . . . .                                    | <br>77 |
| 6.1. Some simple kernels . . . . .  | 77     |
| 6.2. Laplace transforms . . . . .   | 84     |
| 6.3. Solution methods using Laplace transforms . . . . .                          | 86     |
| 6.4. The asymptotic behavior of the solution for some special equations . . . . . | 89     |
| Notes on Chapter 6 . . . . .  | 92     |

### **Part B: Numerical Methods**

|   |         |
|---|---------|
| <br>Chapter 7. The Numerical Solution of Equations of the Second Kind . . . . .       | <br>95  |
| 7.1. A simple numerical procedure . . . . .   | 96      |
| 7.2. Methods based on more accurate numerical integration . . . . .                   | 97      |
| 7.3. Error analysis: convergence of the approximate solution . . . . .                | 100     |
| 7.4. Error estimates and numerical stability . . . . .                                | 103     |
| 7.5. Another view of stability . . . . .  | 110     |
| 7.6. Block-by-block methods . . . . .   | 114     |
| 7.7. Some numerical examples . . . . .  | 118     |
| 7.8. Explicit Runge–Kutta methods . . . . .   | 122     |
| 7.9. A summary of related ideas and methods . . . . .                                 | 124     |
| Notes on Chapter 7 . . . . .  | 127     |
| <br>Chapter 8. Product Integration Methods for Equations of the Second Kind . . . . . | <br>129 |
| 8.1. Product integration . . . . .  | 130     |
| 8.2. A simple method for a specific example . . . . .                                 | 132     |
| 8.3. A method based on the product trapezoidal rule . . . . .                         | 135     |
| 8.4. A block-by-block method based on quadratic interpolation . . . . .               | 136     |
| 8.5. A convergence proof for product integration methods . . . . .                    | 138     |
| Notes on Chapter 8 . . . . .  | 141     |

|   |     |
|---|-----|
| Chapter 9. Equations of the First Kind with Differentiable Kernels . . . . .    | 143 |
| 9.1. Application of simple integration rules . . . . .                          | 144 |
| 9.2. Error analysis for simple approximation methods . . . . .                  | 145 |
| 9.3. Difficulties with higher order methods . . . . .                           | 151 |
| 9.4. Block-by-block methods . . . . .   | 154 |
| 9.5. Use of the differentiated form . . . . .                                   | 158 |
| 9.6. Nonlinear equations . . . . .  | 160 |
| 9.7. Some practical considerations . . . . .                                    | 161 |
| Notes on Chapter 9 . . . . .  | 163 |
| Chapter 10. Equations of the Abel Type . . . . .                                | 165 |
| 10.1. Solving a simple Abel equation . . . . .                                  | 166 |
| 10.2. The midpoint and trapezoidal methods for general Abel equations . . . . . | 166 |
| 10.3. Block-by-block methods . . . . .  | 169 |
| 10.4. Some remarks on error analysis . . . . .                                  | 171 |
| 10.5. Solving Abel equations in the presence of experimental errors . . . . .   | 174 |
| Notes on Chapter 10 . . . . .   | 175 |
| Chapter 11. Integrodifferential Equations . . . . .                             | 177 |
| 11.1. A simple numerical method . . . . .                                       | 178 |
| 11.2. Linear multistep methods . . . . .  | 182 |
| 11.3. Block-by-block methods . . . . .  | 185 |
| 11.4. Numerical stability . . . . .   | 186 |
| 11.5. Other types of integrodifferential and functional equations . . . . .     | 188 |
| Notes on Chapter 11 . . . . .   | 189 |
| Chapter 12. Some Computer Programs . . . . .                                    | 191 |
| 12.1. The trapezoidal method for systems of the second kind . . . . .           | 192 |
| 12.2. The product trapezoidal method for a system of the second kind . . . . .  | 195 |
| 12.3. The midpoint method for systems of the first kind . . . . .               | 197 |
| 12.4. The product midpoint method for a system of the first kind . . . . .      | 198 |
| Notes on Chapter 12 . . . . .   | 199 |
| Chapter 13. Case Studies . . . . .  | 201 |
| 13.1. Estimating errors in the approximation . . . . .                          | 201 |
| 13.2. An example from polymer rheology . . . . .                                | 204 |

|  |     |
|--|-----|
| 13.3. Solving an equation of the first kind in the presence of large data errors . . . . . | 207 |
| Notes on Chapter 13. . . . .   | 211 |
| References . . . . .   | 213 |
| Supplementary Bibliography . . . . .   | 223 |
| Index . . . . .  | 225 |