
Contents

<i>Preface</i>	<i>page</i>	xi
1 The Korteweg–de Vries equation		1
1.1 Preliminaries		1
1.2 The discovery of solitary waves		7
1.3 The discovery of soliton interactions		12
1.4 Applications of the KdV equation		15
Further reading		16
Exercises		17
2 Elementary solutions of the Korteweg–de Vries equation		20
2.1 Travelling-wave solutions		20
2.2 Solitary waves		21
2.3 General waves of permanent form		22
2.4 Description in terms of elliptic functions		26
2.5 Limiting behaviours of the cnoidal wave		29
2.6 Other solutions of the KdV equation		30
Further reading		32
Exercises		33
3 The scattering and inverse scattering problems		39
3.1 Preamble		39
3.2 The scattering problem		40
Example (i): the delta function		44
Example (ii): the sech^2 function		45
3.3 The inverse scattering problem		48
3.4 The solution of the Marchenko equation		56
Example (i): reflection coefficient with one pole		57
Example (ii): zero reflection coefficient		58
Further reading		60
Exercises		61

4 The initial-value problem for the Korteweg–de Vries equation	64
4.1 Recapitulation	64
4.2 Inverse scattering and the KdV equation	65
4.3 Time evolution of the scattering data	67
4.3.1 Discrete spectrum	68
4.3.2 Continuous spectrum	70
4.4 Construction of the solution: summary	71
4.5 Reflectionless potentials	72
Example (i): solitary wave	73
Example (ii): two-soliton solution	74
Example (iii): N -soliton solution	78
4.6 Description of the solution when $b(k) \neq 0$	81
Example (i): delta-function initial profile	81
Example (ii): a negative sech ² initial profile	83
Example (iii): a positive sech ² initial profile	83
Further reading	86
Exercises	86
5 Further properties of the Korteweg–de Vries equation	89
5.1 Conservation laws	89
5.1.1 Introduction	89
5.1.2 An infinity of conservation laws	92
5.1.3 Of Lagrangians and Hamiltonians	95
5.2 Lax formulation and its KdV hierarchy	97
5.2.1 Description of the method: operators	97
5.2.2 The Lax KdV hierarchy	99
5.3 Hirota’s method: the bilinear form	102
5.3.1 The bilinear operator	103
5.3.2 The solution of the bilinear equation	106
5.4 Bäcklund transformations	109
5.4.1 Introductory ideas	110
5.4.2 Bäcklund transformation for the KdV equation	112
5.4.3 The KdV Bäcklund transformation: an algebraic relation	114
5.4.4 Bäcklund transformations and the bilinear form	116
Further reading	118
Exercises	118
6 More general inverse methods	127
6.1 The AKNS scheme	128
6.1.1 The 2×2 eigenvalue problem	128

6.1.2 The inverse scattering problem	131
6.1.3 An example: $r = -q$, $q = \lambda \operatorname{sech} \lambda x$	134
6.1.4 Time evolution of the scattering data	137
6.1.5 The evolution equations for q and r	140
(a) Quadratic in ζ	141
(b) Polynomial in ζ^{-1}	142
(c) General function of ζ	143
6.2 The ZS scheme	144
6.2.1 The integral operators	144
6.2.2 The differential operators	146
6.2.3 Scalar operators	149
(a) The KdV equation	149
(b) The two-dimensional KdV equation	151
6.2.4 Matrix operators	152
(a) The nonlinear Schrödinger equation	152
(b) The sine-Gordon equation	154
6.3 Two examples	157
Example (i): The nonlinear Schrödinger equation	157
Example (ii): The sine-Gordon equation	160
Further reading	162
Exercises	162
7 The Painlevé property, perturbations and numerical methods	169
7.1 The Painlevé property	169
7.1.1 Painlevé equations	169
7.1.2 The Painlevé conjecture	171
7.1.3 Linearisation of the Painlevé equations	172
7.2 Perturbation theory	174
7.2.1 Perturbation theory: an example	177
7.3 Numerical methods	180
7.3.1 Spectral methods	182
7.3.2 Finite-difference methods	183
7.3.3 Long-wave equations	184
7.3.4 Nonlinear Klein-Gordon equations	186
7.3.5 The nonlinear Schrödinger equation	187
Further reading	187
Exercises	187
8 Epilogue	190
8.1 Some numerical solutions of nonlinear evolution equations	190

8.2 Applications of nonlinear evolution equations	196
Further reading	200
Exercises	201
<i>Answers and hints</i>	205
<i>Bibliography and author index</i>	213
<i>Motion picture index</i>	220
<i>Subject index</i>	221