

Contents

Prologue	ix
Chapter 1: The Inverse Scattering Transform on the Infinite Interval	1
1.1. Introduction	1
1.2. Second order eigenvalue problems and related solvable partial differential equations	8
1.3. Derivation of a linear integral equation and inverse scattering on the infinite interval	15
1.4. Time dependence and special solutions	28
1.5. General evolution operator	42
1.6. Conservation laws and complete integrability	52
1.7. Long-time behavior of the solutions	67
Exercises	84
Chapter 2: IST in Other Settings	93
2.1. Higher order eigenvalue problems and multidimensional scattering problems	93
2.2. Discrete problems	114
2.3. Periodic boundary conditions for the Korteweg-deVries equation	134
Exercises	148
Chapter 3: Other Perspectives	151
Overview	151
3.1. Bäcklund transformations	153
3.2. Pseudopotentials and prolongation structures	161
3.3. Direct methods for finding soliton solutions—Hirota’s method	171

3.4.	Rational solutions of nonlinear evolution equations	191
3.5.	N -body problems and nonlinear evolution equations	203
3.6.	Direct approaches with the linear integral equation	217
3.7.	Painlevé transcendents	233
3.8.	Perturbations and transverse stability of solitons and solitary waves	250
	Exercises	261
Chapter 4: Applications		275
4.1.	KdV problems and their cousins	276
4.2.	Three-wave interactions	300
4.3.	The nonlinear Schrödinger equation and generalizations	313
4.4.	Equations of the sine-Gordon type	327
4.5.	Quantum field theory	339
	Exercises	342
Appendix: Linear Problems		351
A.1.	Fourier transforms	351
A.2.	Failure of the Fourier transform method	373
	Exercises	384
Bibliography		393
Index		415