TABLE OF CONTENTS

CHAPTER 1 :		Sequential Solutions of Nonlinear PDEs	1
	0	Introduction	1
	1	The Problem of Stability of Sequential Solutions	2
	2	Defining the Quotient Spaces and Algebras	5
	3	The Notion of Sequential Solution	10
	4	Exactness of Sequential Solutions	12
	5	Interplay between Stability, Generality and Exactness	13
	6	The Deficiency of the Distributional Approach Concerning Exactness	17
	7	Resolution of Nowhere Dense Singularities	19
	8	Preservation of Ellipticity and Hypoellipticity	24
	9	General Nonlinear PDEs	25
	10	Variable Transforms	27
	11	Stronger Conditions for Partial Derivative Operators	30
	12	Systems of Nonlinear PDEs	32
	13	A Cesaro Characterization of the Nowhere Dense Ideal	34
CHAPTER 2 :		Necessary and/or Sufficient Conditions for the Existence of Sequential Solutions	37
	0	Introduction	37
	1	Subsequence Quasi Invariant Sequential Solutions	38
	2	Applications to Linear and Nonlinear PDEs	44
	3	Subsequence Invariant Sequential Solutions	46
	4	Resolvent Sets	51
	5	Domains of Solvability	55
	6	Remarks on Lemma 2	62

CHAPTER 3 : Algebras Containing the Distributions 65

0 Introduction 65

1 Embedding the Distributions into Quotient Algebras 65

2 Simpler Inclusion Diagrams 68

3 Regularizations and Chains of Quotient Algebras Containing the Distributions 69

4 Existence, Characterization and Construction of Regularizations 75

5 Additional Chains of Quotient Algebras Containing Distributions 88

6 Quotient Algebras Containing Large Subspaces of Distributions 97

7 Quotient Algebras Possessing Special Properties 104

8 Definition of Positive Powers for Certain Generalized Functions 107

9 Limitations on Embedding Smooth Functions into Chains of Quotient Algebras 110

10 Special Classes of Regular Ideals 113

11 The Proof of Lemma 1 115

12 Example of Shrinking Non-δ Sequence 117

13 Inexistence of the Largest Regular Ideals 118

CHAPTER 4 : Resolution of Singularities of Weak Solutions for Polynomial Nonlinear PDEs 121

0 Introduction 121

1 The Case of Simple Polynomial Nonlinear PDEs 122

2 Resolution of Singularities of Nonlinear Shock Waves 131

3 Resolution of Singularities of Klein-Gordon Type Nonlinear Waves 133

4 Resolution of Singularities of Weak Solutions for General Polynomial Nonlinear PDEs 134

5 Junction Conditions and Resolution of Singularities of Weak Solutions for the Equations of Magneto-hydrodynamics and General Relativity 139

6 Resoluble Systems of Polynomial Nonlinear PDEs 152

7 Computation of the Junction Conditions 155

8 Examples of Resoluble Systems of PDEs 156

CHAPTER 5: Stability and Exactness of Sequential and Weak Solutions for Polynomial Nonlinear PDEs 163

0 Introduction 163

1 Basic Facts and Remarks Concerning Stability 163

2 Maximal Stability 165

3 Interplay between Stability and Exactness 167

4 Remarks in the Finite Smoothness Case 167

5 Stability of Regular Weak Solutions 168

6 Stability of Solutions with Jump Discontinuities for Resoluble Systems of PDEs 170

CHAPTER 6: Characterization of the Necessary Structure of the Algebras Containing the Distributions 173

0 Introduction 173

1 Zero Filters 174

2 Density Characterization of Z-Filters 176

3 Large C^∞-Regular Ideals 178

4 Further Properties of Z-Filters 182

5 Construction and Characterization of a Class of Vanishing Ideals 185

6 Countable Reduced Products of Vector Spaces 194

CHAPTER 7: Quantum Scattering in Potentials Positive Powers of the Dirac δ Distribution 199

0 Introduction 199

1 Wave Function Solutions and Junction Relations 199

2 Weak Wave Function Solutions 201

3 Smooth Representations for the Dirac δ Distribution and Smooth Weak Wave Function Solutions 210

4 Wave Function Solutions in the Algebras Containing the Distributions 214

5 Stability and Exactness of the Wave Function Solutions 220

CHAPTER 8 : Products with Dirac δ Distributions 223

0 Introduction 223

1 A Class of Regularizations 223

2 Products with Dirac δ Distributions 229

3 Application to a Riccati Differential Equation 234

4 Validity of Formulas in Quantum Mechanics 235

5 The Existence of the Sequences of Functions in Z_δ 242

6 Remark on Nilpotent Elements in the Quotient Algebras 249

CHAPTER 9 : Linear Independent Families of Dirac δ Distributions at a Point 251

0 Introduction 251

1 Compatible Quotient Algebras and Independent Variable Transforms 251

2 Linear Independent Families of Dirac δ Distributions at a Point 255

3 Generalized Dirac δ Distributions 257

CHAPTER 10: Support and Local Properties 265

0 Introduction 265

1 Support of Elements in Quotient Algebras 265

2 Localization within the Quotient Algebras 270

3 Equivalence between S=0 and supp S=∅ 278

4 Domains of Solvability for Polynomial Nonlinear PDEs 279

FINAL REMARKS 281

APPENDIX 1: Neutrix Calculus and Negligible Sequences of Functions 285

APPENDIX 2: The Embedding Impossibility Result of L.Schwartz 289

APPENDIX 3: A Nonlinear Extension of the Lax-Richtmyer Equivalence between Stability and Convergence of

Difference Schemes 293

APPENDIX 4: The Cauchy-Bolzano Quotient Algebra Construction of the Real Numbers 301

REFERENCES 305