

Contents

Preface

ix

I	Dimensional analysis and small parameters	1
1	Dimensional analysis	1
1.1	The main concepts of dimensional analysis	1
1.2	Transformations in dimensional analysis	4
2	Introduction of small parameters	6
2.1	Normalization of equations of motion	6
2.2	Variants of small parameter introduction	14
2.3	Regular and singular perturbations with respect to the small parameter	18
2.4	Two types of power series expansion with respect to a small parameter	21
2.5	Redundancy in methods of approximation	23
II	Regularly perturbed systems. Expansions of solutions	25
3	The Poincaré theorem. The algorithm of expansion	25
4	Applications of the Poincaré theorem	28
4.1	Stokes' problem	28
4.2	Secular terms	30
4.3	Systematic drifts of a gyro in gimbals. Method of successive approximations.	34
5	Poincaré–Lyapunov method	37
5.1	Algorithm of the method	37
5.2	Examples. Nonisochronism of nonlinear system oscillations	39
III	Decomposition of motion in systems with fast phase	43
6	Method of averaging in systems with a single fast phase	44
6.1	Krylov–Bogolyubov equations in standard form	44
6.2	Algorithm of asymptotic expansion	46
6.3	Approximation accuracy	51
6.4	Averaging over trajectories of the generating system	52
6.5	Variants of averaging methods	53

7	Applications of the method of averaging	55
7.1	Free oscillations with friction of various types	55
7.2	Free oscillations of a tube generator	59
8	Method of harmonic linearization	62
8.1	Foundations of the method	62
8.2	Examples.	67
9	Method of averaging in systems with several fast phases	70
9.1	Averaged equations of the first approximation.	70
9.2	Resonances in multifrequency systems	71
9.3	Averaging algorithm in the case of resonance	72
9.4	Pendulum resonance oscillations	74
9.5	Resonant oscillations with friction	79
10	Averaging in systems without explicit periodicities	82
10.1	Volosov averaging scheme	82
10.2	Separation of characteristic motions of an oscillator with high friction	83
IV	Decomposition of motion in systems with boundary layer	87
11	Tikhonov theorem	87
11.1	Introductory considerations	87
11.2	Tikhonov theorem	89
11.3	Decomposition of motion on an infinite time interval	93
12	Application of the Tikhonov theorem	95
12.1	Quasistatic motions of mechanical systems	95
12.2	The method of "frozen coefficients"	98
12.3	The limit model for a double pendulum of high stiffness	100
12.4	Relaxation oscillations of the valve generator	104
13	Asymptotic expansion of solutions for systems with a boundary layer	107
13.1	Algorithm of expansion	107
13.2	Asymptotic expansions for the Stokes problem	113
13.3	Asymptotic expansions on the problem of pendulum motion in a medium of high viscosity	115
13.4	Decomposition of motions of a railway car in magnetic suspension	117
V	Decomposition of motion in systems with discontinuous characteristics	123
14	Definition of a solution in discontinuity points	123
15	Examples	126
15.1	Relay control of angular motion of spacecraft. Sliding mode	126
15.2	Disc rolling motion with Coulomb friction	129
15.3	Relaxation oscillations of the Froude pendulum	133

VI Correctness of limit models	137
16 Limit model of holonomic constraint (absolutely rigid body)	137
16.1 Conditions for correctness of the model in statically definable and indefinable cases	137
16.2 Examples	144
17 Limit model of kinematic constraints	147
17.1 Conditions of model correctness in kinematically definable and indefinable cases	147
17.2 Change of kinematic constraints for rolling of a braked wheel	153
17.3 Kinematic indefinability in a rolling rail car problem	156
18 Limit model of servoconstraint	163
18.1 Conditions of servoconstraint realizability	163
18.2 Realization of servoconstraints, defining the manipulator extremity motion	167
19 Precession and nutation models in gyro theory	173
19.1 Correctness conditions for an extended precession model	173
19.2 Precession model for a gyrotachometer	178
19.3 Precession model of a three-axis force gyrostabilizer	179
19.4 Two-step method for stability approval of the nutation model for a three-axis gyrostabilizer	183
20 Mathematical model of a "man-artificial-kidney" system	186
21 Approximate models of an aircraft motion	194
21.1 Models of zeroth approximation with respect to small parameters	194
21.2 Refined models of motion	201
22 Automobile motion decomposition	206
22.1 Structure of automobile motion partial models . . .	207
22.2 Mathematical model of rolling for a deformed wheel. Are nonlinear nonholonomic constraints possible? . .	211
<i>References</i>	<i>223</i>
<i>Author Index</i>	<i>229</i>
<i>Subject Index</i>	<i>230</i>