

# Contents

Preface	xi
Chapter 1. Introduction	1
1.1. Thermodynamic formalism for dynamical systems with chaotic behavior	1
1.2. Main notions of smooth hyperbolic theory and symbolic dynamics	4
1.3. Dynamical systems with singularities	6
1.4. The Li–Yorke chaos	8
1.5. Measurable time change in chaotic dynamical systems	9
Chapter 2. Operator Approach in Chaotic Dynamics	13
2.1. The Perron–Frobenius operator and its properties	13
2.2. Abstract ergodic theorem of Ionescu–Tulcea and Marinescu	13
2.3. Functions of bounded generalized variation	20
2.3.1. Bounded variation in sense of distributions	31
2.3.2. Functions of bounded variation on discrete lattices	31
2.4. Piecewise expanding maps	34
2.5. Conjugation of a general one-dimensional PE map to a uniformly expanding one	36
2.6. Counterexamples	38
2.7. Conditionally invariant measures and marginal singularities	41
2.8. Finite dimensional stochastic attractors of certain infinite dimensional systems	46
Chapter 3. Random Perturbations of Dynamical Systems	53
3.1. Random perturbations of smooth hyperbolic systems. Stability of stochastic attractors (review)	53
3.2. Random perturbations of systems with singularities. Stability of stochastic attractors	56
3.3. Counterexamples. Stabilization of singular invariant measures (localization)	59
3.4. Random perturbations of general one-dimensional piecewise expanding maps	62
3.4.1. The deterministic part of the dynamics	64
3.4.2. The stochastic part of the dynamics	65
3.4.3. The decomposition	66
3.4.4. The scheme of the proofs	67
3.4.5. Estimates far from turning points. Interchanging the map and the perturbation	67
3.4.6. Estimates close to turning points	72

3.4.7. Discussion of technical assumptions	79
3.5. The most probable trajectories of a dynamical system	82
Chapter 4. Weakly Coupled Dynamical Systems	95
4.1. Definitions and main problems	95
4.2. Operator approach to CML. Generic case.	98
4.3. Operator approach to CML. Localization for the case with periodic turning points.	102
4.4. Localization in CML constructed using smooth maps	107
Chapter 5. Phase Space Discretization in Dynamical Systems	109
5.1. Definitions and main examples	109
5.2. Properties of periodic trajectories under discretization	110
5.3. Statistical probability for discretized systems	115
5.4. Case of neutral periodic trajectories	118
5.5. A KAM type theorem for systems with round-off errors	124
5.5.1. Twist maps with $C^\infty$ -smooth perturbations	125
5.5.2. Twist maps with discontinuous perturbations	127
5.6. Relative volume of the maximal invariant set of a discretized system	131
5.7. Partial space discretizations	133
5.8. A more detailed model of round-off	134
Chapter 6. Ergodic Properties of Some Methods for Numerical Modeling of Chaotic Dynamics	139
6.1. Numerical approximation of invariant measures	139
6.1.1. Numerical method based on a specific random perturbation	139
6.1.2. Ulam's procedure	144
6.2. Shadowing of individual trajectories	147
6.3. Shadowing on average for on average small perturbations	151
Bibliography	157
Index	161